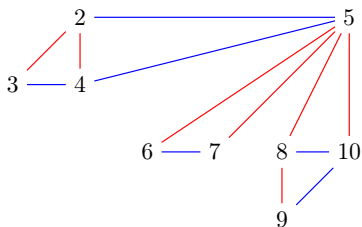


Involution “Montées - Contacts” sur les intervalles de Tamari

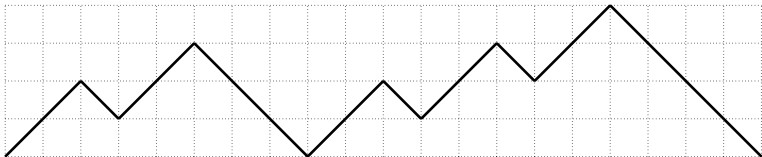
Viviane Pons
Université Paris-Saclay

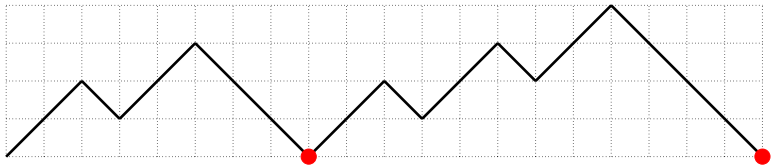
1



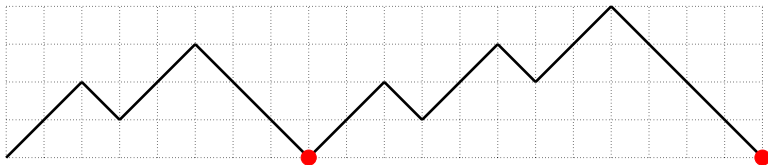
Motivations

- ▶ M. Bousquet-Mélou, E. Fusy, and L.-F. Préville-Ratelle. The number of intervals in the m -Tamari lattices. *Electron. J. Combin.*, 2011.
- ▶ L.-F. Préville-Ratelle. *Combinatoire des espaces coinvariants trivariés du groupe symétrique*. Thèse de Doctorat, Université du Québec à Montréal, 2012.
- ▶ F. Bergeron and L.-F. Préville-Ratelle. Higher trivariate diagonal harmonics via generalized Tamari posets. *J. Comb.*, 2012.



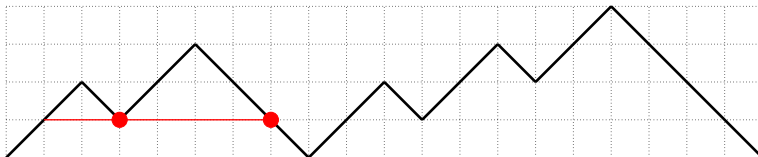


Contacts : 2



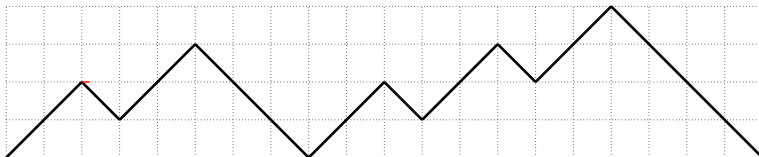
Contacts : 2

Contact vector: 2,



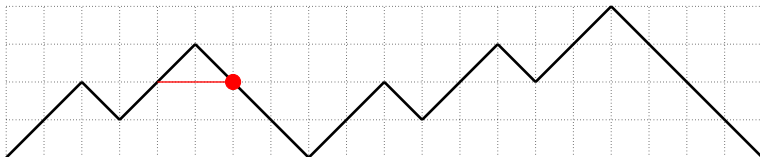
Contacts : 2

Contact vector: 2, 2,



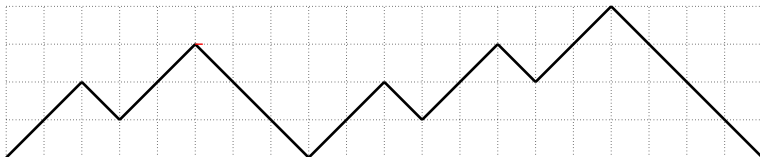
Contacts : 2

Contact vector: 2, 2, 0,



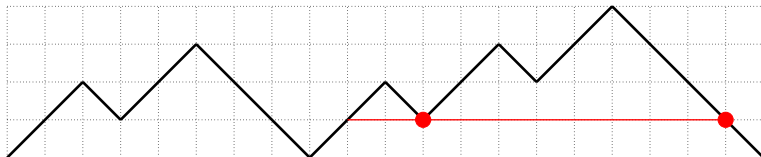
Contacts : 2

Contact vector: 2, 2, 0, 1,



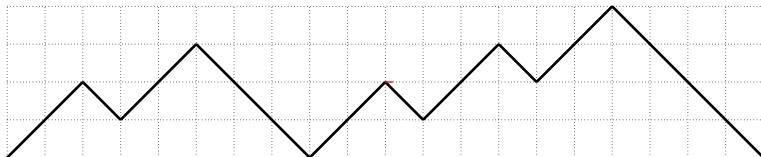
Contacts : 2

Contact vector: 2, 2, 0, 1, 0,



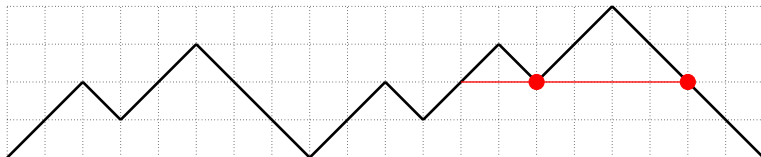
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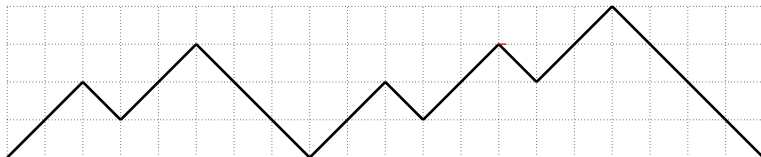
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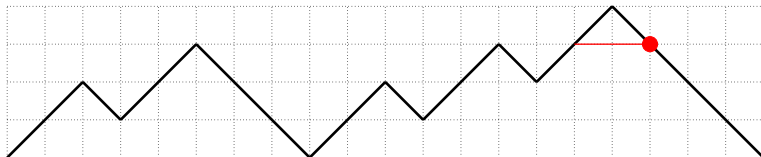
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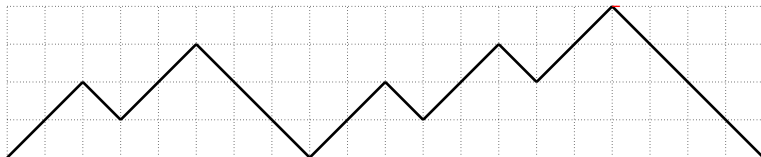
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Contact vector: 2, 2, 0, 1, 0, 2, 0, 2, 0,



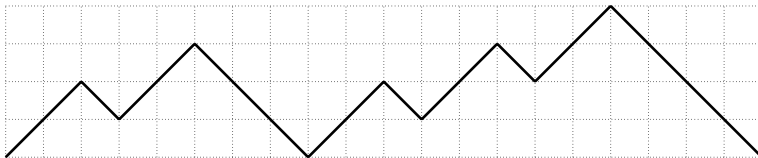
Contacts : 2

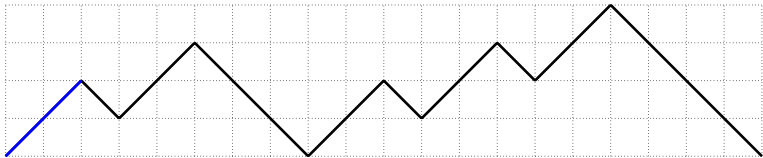
Contact vector: 2, 2, 0, 1, 0, 2, 0, 2, 0, 1,



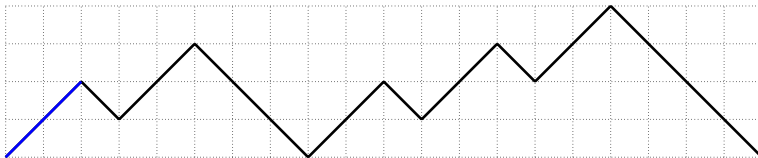
Contacts : 2

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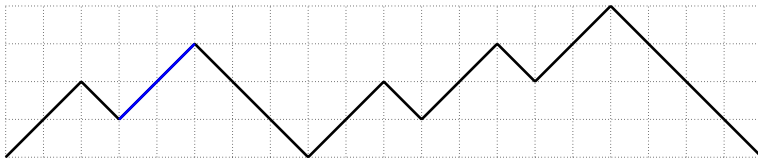


Montées : 2



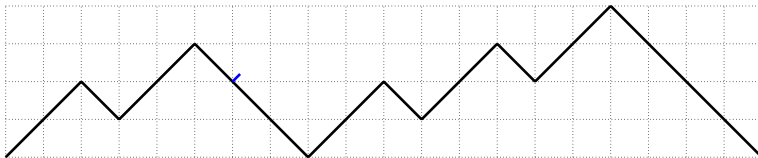
Montées : 2

Rises vector: 2,



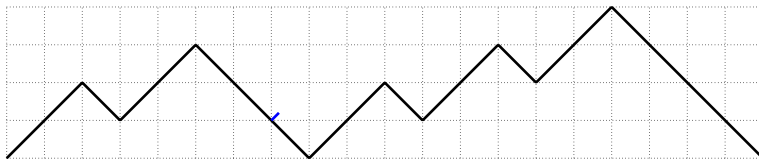
Montées : 2

Rises vector: 2, 2,



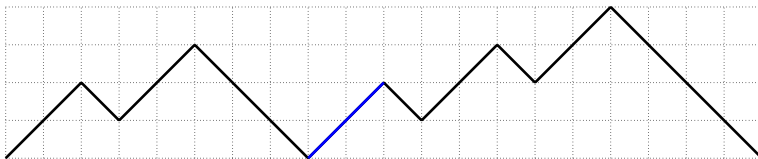
Montées : 2

Rises vector: 2, 2, 0,



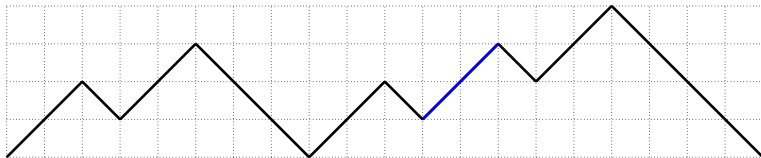
Montées : 2

Rises vector: 2, 2, 0, 0,



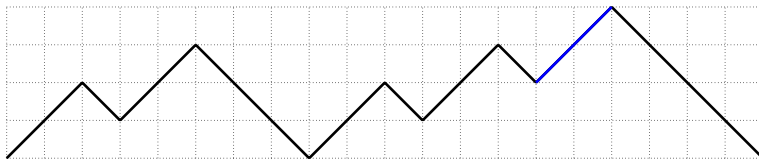
Montées : 2

Rises vector: 2, 2, 0, 0, 2,



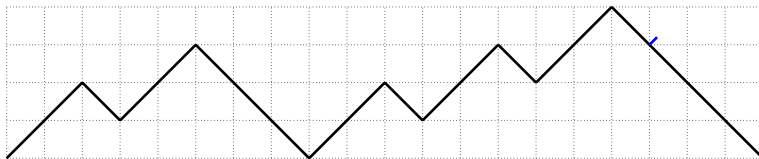
Montées : 2

Rises vector: 2, 2, 0, 0, 2, 2,



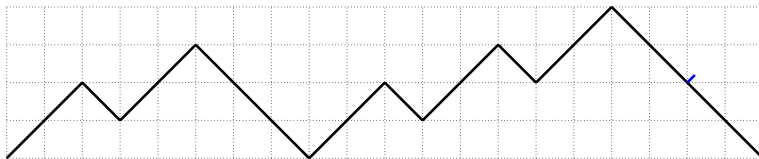
Montées : 2

Rises vector: 2, 2, 0, 0, 2, 2, 2,



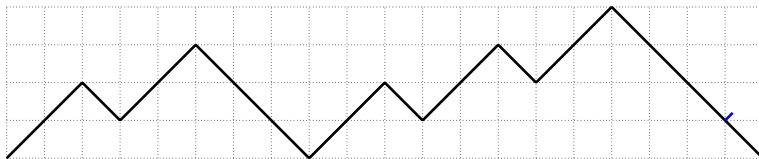
Montées : 2

Rises vector: 2, 2, 0, 0, 2, 2, 2, 0,



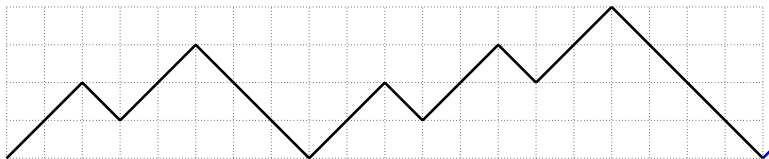
Montées : 2

Rises vector: 2, 2, 0, 0, 2, 2, 2, 0, 0,



Montées : 2

Rises vector: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0,

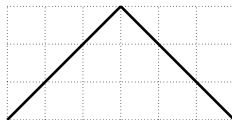


Montées : 2

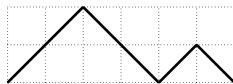
Rises vector: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0



3,0,0 1,1,1



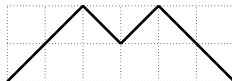
1,1,1 3,0,0



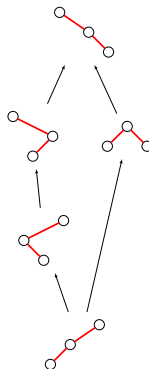
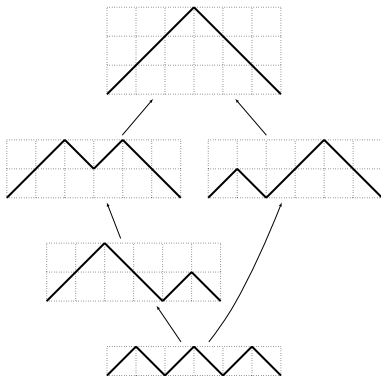
2,1,0 2,0,1

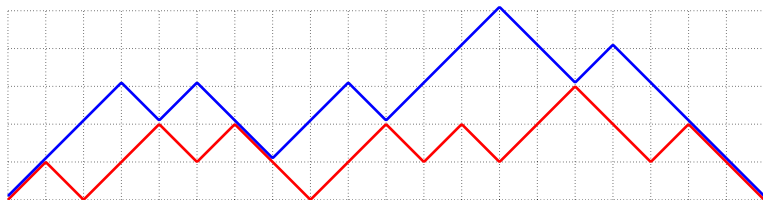


2,0,1 1,2,0



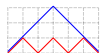
1,2,0 2,1,0



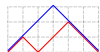


Contact vector: 3, 0, 2, 0, 0, 4, 0, 0, 1, 0

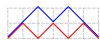
Rise vector: 3, 1, 0, 2, 3, 0, 1, 0, 0, 0



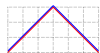
3, 0, 0 3, 0, 0



2, 0, 1 3, 0, 0 2, 0, 1



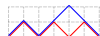
3, 0, 0 2, 1, 0 2, 1, 0 3, 0, 0



1, 1, 1 3, 0, 0 3, 0, 0 1, 1, 1



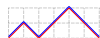
1, 2, 0 3, 0, 0 3, 0, 0 1, 2, 0



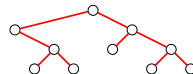
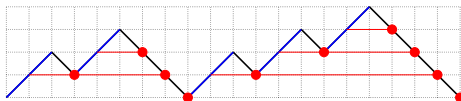
2, 1, 0 2, 1, 0



1, 2, 0 2, 1, 0 2, 0, 1 1, 2, 0

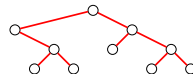
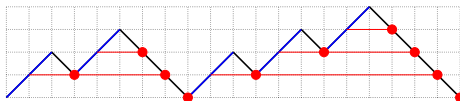


2, 1, 0 2, 0, 1



Contact vector: 2, 2, 0, 1, 0, 2, 0, 2, 0, 1

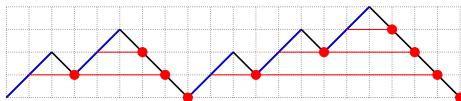
Rise vector: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0



Contact vector: 2, 2, 0, 1, 0, 2, 0, 2, 0, 1

Rise vector: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0

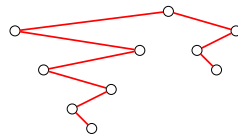
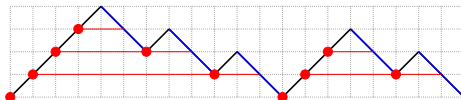
$\downarrow \Phi$



Contact vector: 2, 2, 0, 1, 0, 2, 0, 2, 0, 1

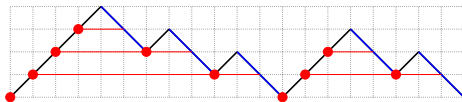
Rise vector: 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 0

$\downarrow \Phi$



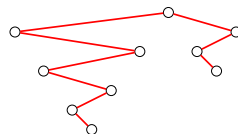
Contact vector: 2, 2, 2, 1, 0, 0, 0, 2, 1, 0

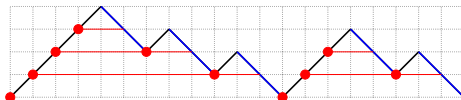
rotcev esiR: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0



Contact vector: 2, 2, 2, 1, 0, 0, 0, 2, 1, 0

rotcev esiR: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0

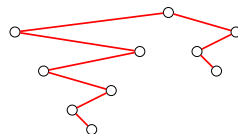


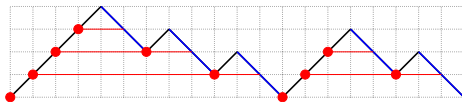


Contact vector: 2, 2, 2, 1, 0, 0, 0, 2, 1, 0

rotcev esiR: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0

$\downarrow \psi$

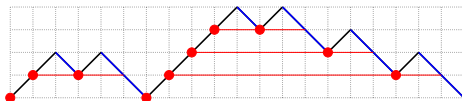




Contact vector: 2, 2, 2, 1, 0, 0, 0, 2, 1, 0

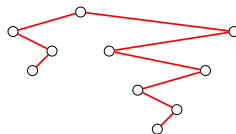
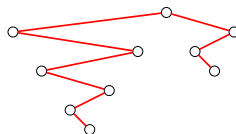
rotcev esiR: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0

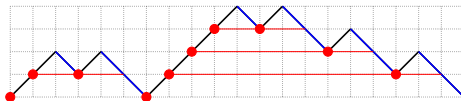
$\downarrow \psi$



Contact vector: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0

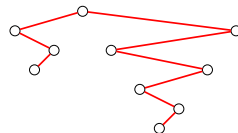
rotcev esiR: 2, 2, 2, 1, 0, 0, 0, 2, 1, 0

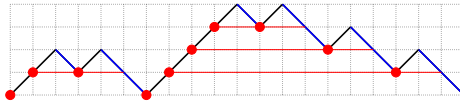




Contact vector: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0

rotcev esiR: 2, 2, 2, 1, 0, 0, 0, 2, 1, 0

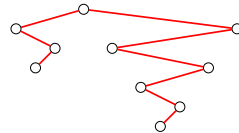


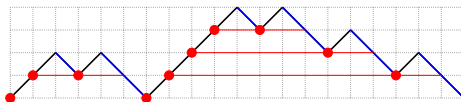


Contact vector: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0

rotcev esiR: 2, 2, 2, 1, 0, 0, 0, 2, 1, 0

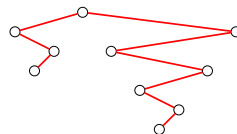
$\downarrow \Phi$



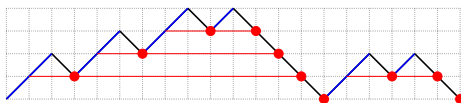


Contact vector: 2, 2, 0, 0, 2, 2, 2, 0, 0, 0

rotcev esiR: 2, 2, 2, 1, 0, 0, 0, 2, 1, 0

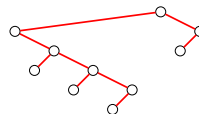


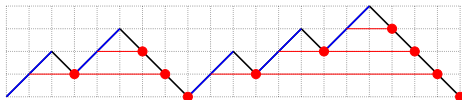
$\downarrow \Phi$



Contact vector: 2, 2, 0, 2, 0, 2, 0, 0, 2, 0

Rise vector: 2, 2, 2, 1, 0, 0, 0, 2, 1, 0

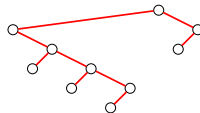
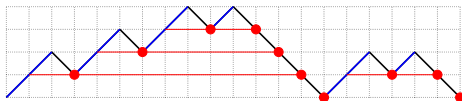




Contact vector: $2, 2, 0, 1, 0, 2, 0, 2, 0, 1 - \underline{2^4, 1^2, 0^4}$

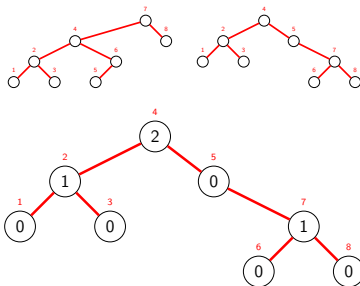
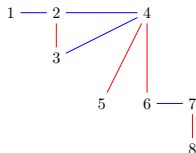
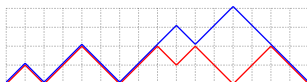
Rise vector: $2, 2, 0, 0, 2, 2, 2, 0, 0, 0 - \underline{2^5, 0^5}$

$\downarrow \Phi \circ \Psi \circ \Phi$



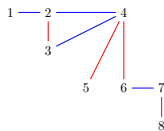
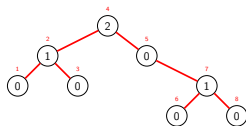
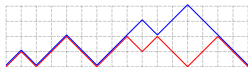
Contact vector: $2, 2, 0, 2, 0, 2, 0, 0, 2, 0 - \underline{2^5, 0^5}$

Rise vector: $2, 2, 2, 1, 0, 0, 0, 2, 1, 0 - \underline{2^4, 1^2, 0^4}$



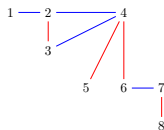
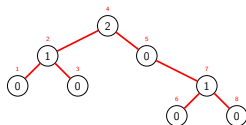
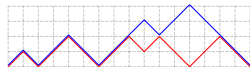
Contact vector: 4, 0, 1, 0, 2, 0, 0, 1

Rise vector: 1, 2, 0, 3, 2, 0, 0, 0



Contact vector: 4, 0, 1, 0, 2, 0, 0, 1

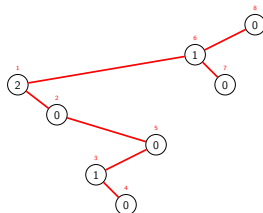
Rise vector: 1, 2, 0, 3, 2, 0, 0, 0

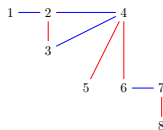
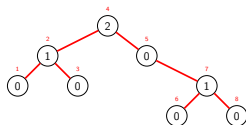
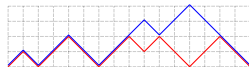


Contact vector: 4, 0, 1, 0, 2, 0, 0, 1

Rise vector: 1, 2, 0, 3, 2, 0, 0, 0

$\downarrow \Phi$

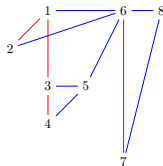
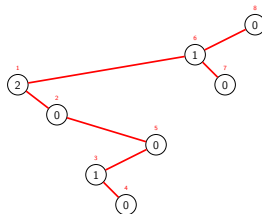




Contact vector: 4, 0, 1, 0, 2, 0, 0, 1

Rise vector: 1, 2, 0, 3, 2, 0, 0, 0

$\downarrow \Phi$



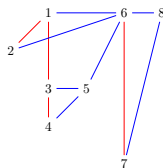
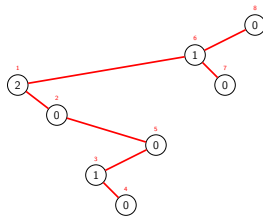
Contact vector: 4, 2, 0, 1, 0, 0, 1, 0

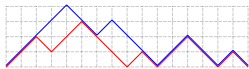
rotcev esiR: 1, 2, 0, 3, 2, 0, 0, 0



Contact vector: 4, 2, 0, 1, 0, 0, 1, 0

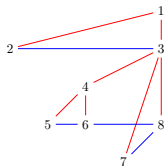
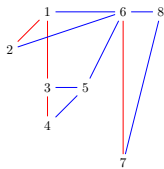
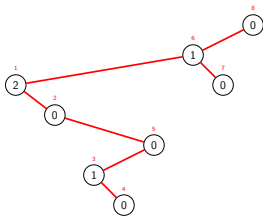
rotcev esiR: 1, 2, 0, 3, 2, 0, 0, 0





rotcev esiR: 1, 2, 0, 3, 2, 0, 0, 0

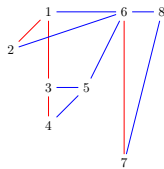
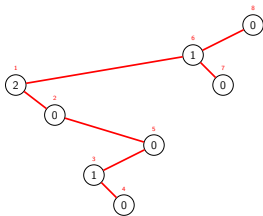
↓ Ψ



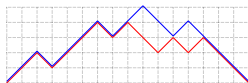


Contact vector: 4, 2, 0, 1, 0, 0, 1, 0

rotcev esiR: 1, 2, 0, 3, 2, 0, 0, 0

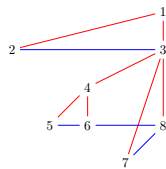
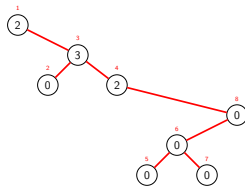


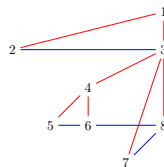
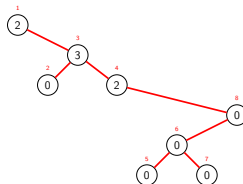
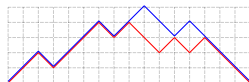
$\downarrow \Psi$



Contact vector: 1, 2, 0, 3, 2, 0, 0, 0

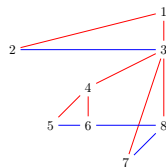
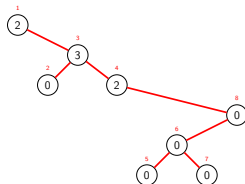
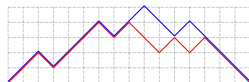
rotcev esiR: 4, 2, 0, 1, 0, 0, 1, 0





Contact vector: 1, 2, 0, 3, 2, 0, 0, 0

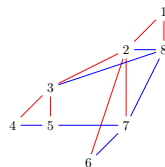
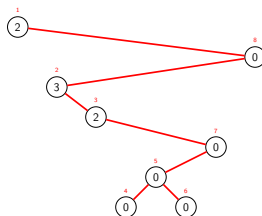
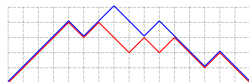
rotcev esiR: 4, 2, 0, 1, 0, 0, 1, 0



Contact vector: 1, 2, 0, 3, 2, 0, 0, 0

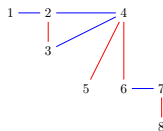
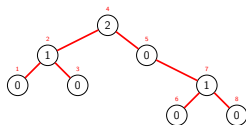
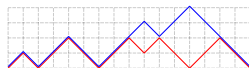
rotcev esiR: 4, 2, 0, 1, 0, 0, 1, 0

$\downarrow \Phi$



Contact vector: 1, 2, 3, 2, 0, 0, 0, 0

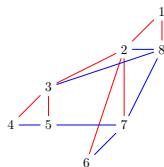
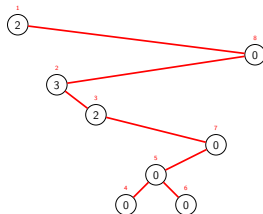
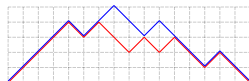
Rise vector: 4, 2, 0, 1, 0, 0, 1, 0



Contact vector: 4, 0, 1, 0, 2, 0, 0, 1 – 4, 2, 1², 0⁴

Rise vector: 1, 2, 0, 3, 2, 0, 0, 0 – 3, 2², 1, 0⁴

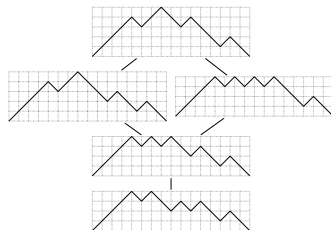
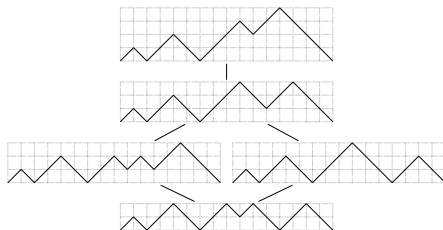
$\downarrow \Phi \circ \Psi \circ \Phi$



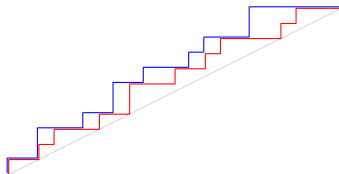
Contact vector: 1, 2, 3, 2, 0, 0, 0, 0 – 3, 2², 1, 0⁴

Rise vector: 4, 2, 0, 1, 0, 0, 1, 0 – 4, 2, 1², 0⁴

Conserve la "distance"



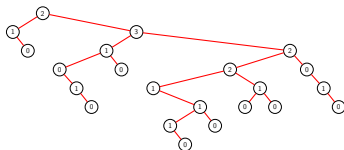
Généralisation à m -Tamari



m -Contact vector: 5, 0, 0, 1, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0

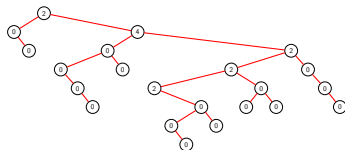
m -Rise vector: 1, 0, 2, 0, 0, 1, 0, 2, 0, 1, 0, 0, 1, 1, 0, 0, 2, 0, 0, 0, 0

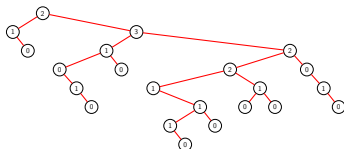




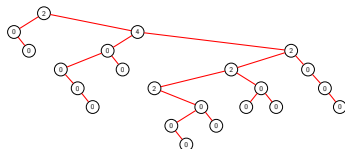
expand $\rightarrow$

$\downarrow \Phi \circ \Psi \circ \Phi$

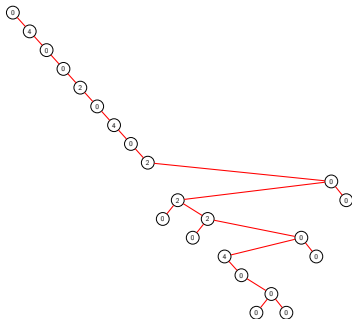




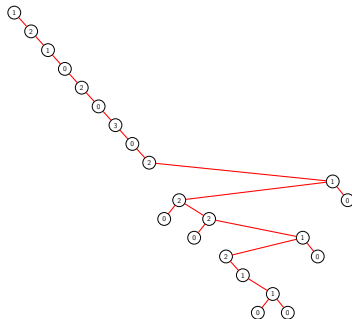
expand $\rightarrow$

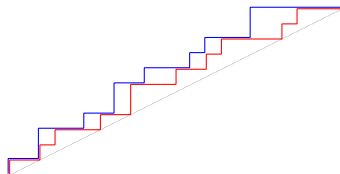


$\downarrow \Phi \circ \Psi \circ \Phi$



contract $\rightarrow$

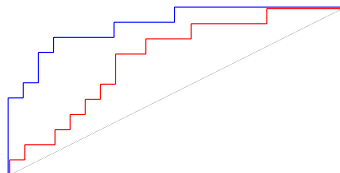




m-Contact vector: 5, 0, 0, 1, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0 – 5, 2, 1⁴, 0¹⁶

m-Rise vector: 1, 0, 2, 0, 0, 1, 0, 2, 0, 1, 0, 0, 1, 1, 0, 0, 2, 0, 0, 0, 0, 0 – 2³, 1⁵, 0¹⁴

expand $\rightarrow \Phi \circ \Psi \circ \Phi \rightarrow$ contract



m-Contact vector: 1, 2, 0, 0, 0, 1, 0, 2, 0, 1, 0, 1, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0 – 2³, 1⁵, 0¹⁴

m-Rise vector: 5, 1, 2, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 – 5, 2, 1⁴, 0¹⁶

Merci !

- ▶ **Article** : The Rise-Contact involution on Tamari intervals.
The Electronic Journal of Combinatorics, 26(2):P2.32, 2019.
arXiv:1701.07995
- ▶ **Démo SageMath** :
github.com/VivianePons/public-notebooks/