Defenses Festival

17-20 October 2023

Hour	Tuesday 17/10	Wednesday 18/10	Thursday 19/10	Friday 20/10
10am		Noémie CARTIER Salle thèses, Bât 650 Univ. Paris-Saclay		
3pm	Daniel TAMAYO JIMÉNEZ Amphi. Bât 660 Univ. Paris-Saclay			Viviane PONS Salle thèses, Bât 650 Univ. Paris-Saclay
5pm		Balthazar CHARLES Salle thèses, Bât 650 Univ. Paris-Saclay	Germain POULLOT Jussieu (15/16.101)	

Here are the 4 Doctoral Theses and the Habilitation to Direct Research that we are presenting to you this week. Above, you will find the schedule and the defense rooms. In the following pages, we offer you a summary of each of the theses/HDR of this week.

Enjoy reading, and please feel free to reach out if you would like to read our theses in detail or if you have any questions!

Daniel TAMAYO JIMÉNEZ

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Ph.D. thesis

Team
GALaC (LISN)

Advisors
Viviane PONS
Vincent PILAUD

Defense on
17 October 2023, 3pm
at Amphithéâtre Bât 660
(Digiteo) Univ. Paris-Saclay

Referees
Samuele GIRAUDO
Torsten MUTZE

Jury
Nathalie AUBRUN
Mathilde BOUVEL
Jean-Christophe NOVELLI
Francisco SANTOS LEAL

After my thesis
Transitioning to industry in Île-de-France

Combinatorics of Permutreehedra and Geometry of s-Permutahedra

Abstract for everyone: In algebraic combinatorics, lattices are partially ordered sets which possess both meet and join operations. The weak order on permutations is a classical example of a lattice that has a rich combinatorial structure. This has made it a starting point from which other combinatorial objects have been defined. For this thesis, we focus on studying two different families of lattices in relation to the weak order: the permutree lattices and the s-weak order.

The first part of the thesis involves the theory of lattice quotients of the weak order building upon the work of N. Reading, specifically focusing on the family of permutree quotients of the weak order. Considering them as permutrees, as done by V. Pilaud and V. Pons, we extend the technology of bracket vectors from binary trees by defining inversion and cubic vectors. The inversion vector captures the meet operation of these lattices while the cubic vector helps realize them geometrically via a cubical embedding. Changing our point of view and studying these quotients through the minimal elements of their congruence classes, we use the Coxeter Type A description of permutations to characterize permutrees using automata. These automata capture the pattern avoidance of the patterns ijk and/or kij implied by these quotients and allow us to define algorithms which generalize stack sorting. In the case where the quotient corresponds to a Cambrian lattice we relate our automata with Coxeter sorting. We give some insight about the same phenomenon for Coxeter groups of types B and D.

The second part of this thesis stems from the work of V. Pons and C. Ceballos who defined the s-weak order on s-decreasing trees where s is a sequence of non-negative integers. In the case of $s = (1, \dots, 1)$ this definition recovers the weak order. In their first article, the authors conjectured that the s-permutahedron could be realized in space as a polyhedral subdivision of a zonotope. We give a positive answer to their conjecture when s is a sequence of positive integers by defining a graph whose flow polytope allows us to recover the s-weak order. We use techniques from flows on graphs, discrete geometry, and tropical geometry to obtain realizations of the s-permutahedron with different properties. Finally, we introduce a graph operation to describe permutrees and their lattices through flow polytopes.

Noémie CARTIER

Ph.D. thesis noemie.m.cartier@gmail.com

Team
My team

Advisors
Florent HIVERT
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Defense on
18 October 2023, 10am
at Salle 435, Bât 650,
LISN, Univ. Paris-Saclay

Referees
Jean FROMENTIN
Christian STUMP

Jury
Emily BARNARD
Matthieu JOSUAT-VERGÈS
Pierre-Guy PLAMONDON
Viviane PONS

After my thesis
my after

Lattices properties of acyclic pipe dreams

Abstract for specialists: This thesis comes within the scope of algebraic combinatorics. Some sorting algorithms can be described by diagrams called sorting networks, and the execution of the algorithms on input permutations translates to arrangements of curves on the networks. These arrangements modelize some classical combinatorial structures: for example, the Tamari lattice, whose cover relations are the rotations on binary trees, and which is a well-known quotient of the weak order on permutations.

Subword complexes generalize sorting network and arrangements of curves to Coxeter groups. They have deep connections in algebra and geometry, in particular in Schubert calculus, in the study of grassmannian varieties, and in the theory of cluster algebras. This thesis focuses on lattice structures on some subword complexes, generalizing Tamari lattices. More precisely, it studies the relation defined by linear extensions of the facets of a subword complex.

At first we focus on subword complexes defined on a triangular word of the symmetric group, which we represent with triangular pipe dreams. We prove that this relation defines a lattice quotient of a weak order interval; moreover, we can also use this relation to define a lattice morphism from this interval to the restriction of the flip graph of the subword complex to some of its facets. Secondly, we extent our study to subword complexes defined on alternating words of the symmetric group. We prove that this same relation also defines a lattice quotient; however, the image of the associated morphism is no longer the flip graph, but the skeleton of the brick polyhedron, an object defines on subword complexes to study realizations of the multiassociahedron. Finally, we discuss possible extensions of these results to finite Coxeter groups, as well as their applications to generalize some objects defined in type A such as nu-Tamari lattices.

Balthazar CHARLES

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Ph.D. thesis

Team
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Advisor
Nicolas THIÉRY

Defense on
18 October 2023, 5pm

Referees
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James MITCHELL

Jury
Lila BOUKHATEM
Suzanna FISCHEL
Vincent PILAUD
Vic REINER
Benjamin STEINBERG

After my thesis
Just sadness

Combinatorics and Computations: Cartan matrices of monoids & Minimal elements of Shi arrangements

Abstract for everyone: This thesis presents an investigation into two distinct combinatorial subjects: the effective computation of Cartan matrices in monoid representation theory and the exploration of properties of minimal elements in Shi arrangements of Coxeter groups. Although disparate, both of these research focuses share a commonality in the utilization of combinatorial methods and computer exploration either as an end in itself for the former or as a help to research for the latter.

In the first part of the dissertation, we develop methods for the effective computation of character tables and Cartan matrices in monoid representation theory. To this end, we present an algorithm based on our results for the efficient computations of fixed points under a conjugacy-like action, with the goal to implement Thiéry's formula for the Cartan matrix from [Thiéry '12]. Based on our fixed-point counting method, as well as a new formula for the character table of finite monoids, we give algorithms for the computation of these invariants. In terms of execution time and memory usage we measure that these methods are more efficient than algorithms not specialized for monoids by orders of magnitude. We hope that the resulting (public) implementation will contribute to the monoid representation community by allowing previously impractical computations.

The second part of the thesis focuses on the properties of minimal elements in Shi arrangements. The Shi arrangements were introduced in [Shi '87] and are the object of Conjecture 2 from [Dyer, Hohlweg '14]. Originally motivated by this conjecture, we present two results. Firstly, a direct proof in the case of rank 3 groups. Secondly, in the special case of Weyl groups, we give a description of the minimal elements of the Shi regions by extending a bijection from [Athanasiadis, Linusson '99] and [Armstrong, Reiner, Rhoades '15] between parking functions and Shi regions. This allows for the effective computation of the minimal elements. From the properties of this computation, we provide a type-free proof of the conjecture in Weyl groups as an application. These results reveal an intriguing interplay between the non-nesting and non-crossing worlds in the case of classical Weyl groups.

Germain POULLOT germain.poullot@imj-prg.fr

Ph.D. thesis

Team
Combinatorics and Optimization
Jussieu

Advisors
Arnau PADROL
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Defense on
19 October 2023, 5pm
at Jussieu (15/16.101)

Referees
Fu LIU
Lionel POURNIN

Jury
Jesús DE LOERA
Martina JUHNKE
Frédéric MEUNIER
Vic REINER

After my thesis
Post-doc (3 years)
at Osnabrück (Germany)
with Martina JUHNKE

Geometric combinatorics of paths and deformations of convex polytopes

Abstract for everyone: From the five Platonic solids to the Grande Arche de la Défense and Renaissance paintings, polyhedra have appeared throughout the history of Arts and Letters, playing a prominent role in mathematics. They remain ubiquitous today in fields like biology (virus' capsids), chemistry (crystallography), and modeling (finite elements method), among others. Their practical applications are numerous, from furniture and video game character design to origami and packaging...

In this thesis, I delve into the study of polytopes, which generalize polyhedra. I explore their combinatorial properties, building upon previous work on the "permutahedron" whose vertices correspond to permutations of the numbers from 1 to n, and whose deformations reveal rich combinatorial structures. I also focus on the geometric aspects of polytopes through linearly constrained optimization, aiming to establish connections between existing research on pivot rule polytopes and deformations of the permutahedron.

Abstract for specialists: The main objects of the present manuscript are polytopes: a polytope is defined as the convex hull of finitely many points in the Euclidean space $\mathbb{R}^d$. As such, polytopes are the generalization of polygons and polyhedra to higher dimensions. In this thesis, I will try to unveil some links between the geometric aspects of polytopes and their combinatorial behaviors. Two concepts will be at the center of this polytopal journey: generalized permutahedra and linear programs.

The first notion arises from the systematic research of the combinatorial properties of polytopes, which have played a great role in the development of the field since their (re)popularization during the 20th century. Polytopes naturally come with various combinatorial properties: foremost, one can try to understand their faces (which are themselves polytopes), and how its faces are included one in another, leading to the definition of the face lattice of a polytopes. If exploring the face lattice of a polytope is already fascinating, the reverse question turns out to be even more fecund: given a combinatorial structure, how to construct a polytope to embody it? An epitome of such quest is surely the construction of the permutahedron. Discovered by Schoute in 1911, the vertices of the permutahedron are in one-to-one correspondence with the permutations. Moreover, the faces of it can be labelled by the ordered partitions, while its (oriented) graph naturally describes the Bruhat order on permutations. But this is only the tip of the iceberg: the permutahedron can be deformed to create generalized permutahedra. Originally defined by Edmonds under the name polymatroids, their rediscovery by Postnikov in 2009 was the starting point of a myriad of researches. In particular, various combinatorial families can be encapsulated in the combinatorics of certain generalized permutahedra.

On the other side, linear programming delve into the geometrical aspects of polytopes. Optimization is known for being a supremely useful but notably difficult theory, and linear optimization encompasses the optimization problems in which both the constraints and the quantity to optimize are linear in the involved variables. There are several methods to solve a linear problem, among which some are known to be of polynomial complexity, but the original method, which is still of prime importance, is the simplex method, whose complexity class is not fully understood for now. The simplex method can be thought of as the counterpart of the Gaussian elimination, but when dealing with linear inequalities (and a linear functional to optimize). In broad, the key idea is to consider the set of solutions of your system of inequalities as a polytope (or an unbounded polyhedra), and to jump from one vertex onto one of its neighbor, increasing the value of the linear functional at each step. Nevertheless, one needs to set up a rule on how to choose the neighbor to jump onto: this is the pivot rule. Pivot rules and how to elect the right one have been written about extensively, and we certainly do not intend to fully answer this question here.

In this manuscript, we study on the one hand generalized permutahedra and the submodular cone, and on the other hand max-slope pivot rule polytopes and fiber polytopes. Although the domains undeniably interact all along the present thesis, ideas coming from one side being steadily applied to the other, the pre-eminent result creating a neat bridge between these two realm is Section 3.3: we state that the combinatorial behavior of the class of shadow vertex rules can be handled more easily by embedding the question inside the realm of generalized permutahedra. We hope that such a new insight may open the way to a better understanding of (memory-less) pivot rules.

Viviane PONS

Habilitation à diriger des recherches

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Team
GALaC, LISN

Defense on
20 October 2023, 3pm

Referees
Sylvie CORTEEL
Nathan READING
Vic REINER

Jury
Mireille BOUSQUET-MÉLOU
Florent HIVERT
Lionel POURNIN
Maria RONCO

Combinatorics of the Permutahedra, Associahedra, and Friends

Abstract for everyone: I present an overview of the research I have conducted for the past ten years in algebraic, bijective, enumerative, and geometric combinatorics. The two main objects I have studied are the *permutahedron* and the *associahedron* as well as the two partial orders they are related to: the weak order on permutations and the Tamari lattice. The document contains a general introduction (Chapters 1 and 2) on those objects which requires very little previous knowledge and should be accessible to non-specialist such as master students. Chapters 3 to 8 present the research I have conducted and its general context. You will find:

- a presentation of the current knowledge on *Tamari intervals* and a precise description of the family of *Tamari interval-posets* which I have introduced along with the *rise-contact involution* to prove the symmetry of the *rises* and the *contacts* in Tamari intervals;
- my most recent results concerning q,t-enumeration of Catalan objects and Tamari intervals in relation with *triangular partitions*;
- the descriptions of the *integer poset lattice* and *integer poset Hopf algebra* and their relations to well known structures in algebraic combinatorics;
- the construction of the *permutree lattice*, the *permutree Hopf algebra* and *permutreehedron*;
- the construction of the *s-weak order* and *s-permutahedron* along with the *s-Tamari lattice* and *s-associahedron*.

Chapter 9 is dedicated to the *experimental method* in combinatorics research especially related to the `SageMath` software. Chapter 10 describes the outreach efforts I have participated in and some of my approach towards mathematical knowledge and inclusion.