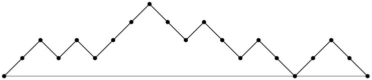
Réalisation de m-Tamari sur les arbres

Viviane Pons

Universität Wien

LIX, 25/06/2014

Chemins de Dyck

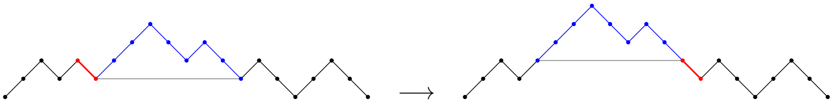


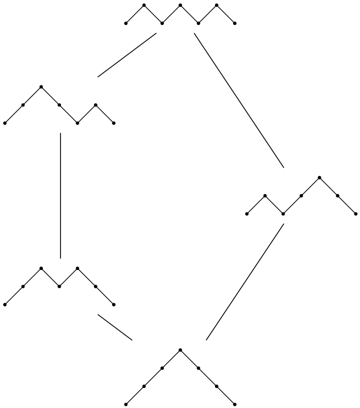
Comptés par les nombres de Catalan :

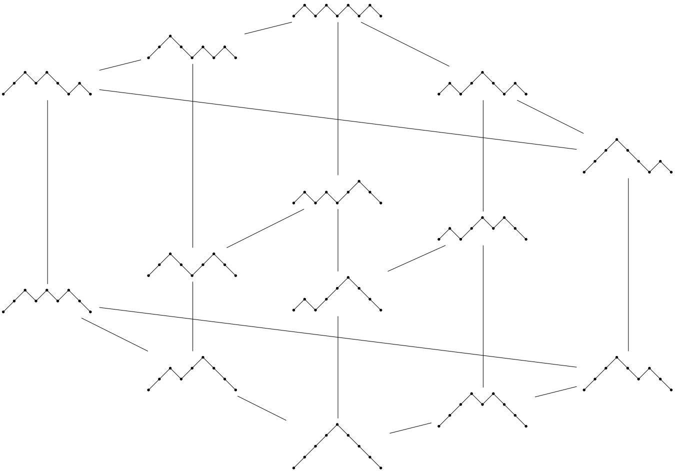
$$\frac{1}{n+1}\binom{2n}{n}$$

Treillis de Tamari sur les chemins de Dyck

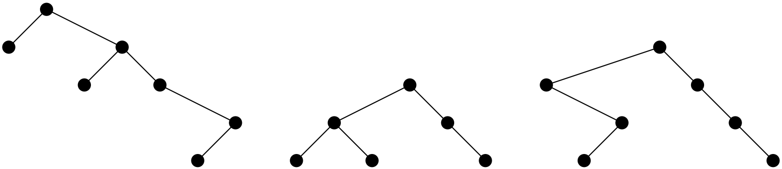
relation de couverture : rotation



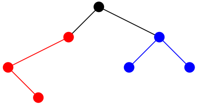




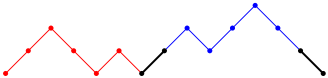
Arbres binaires



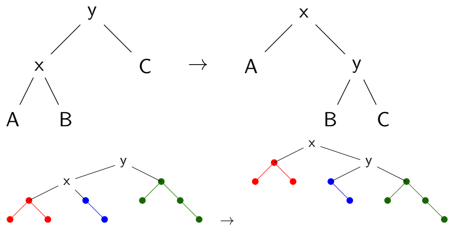
Bijection arbres - chemins

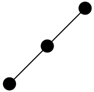


$\longleftrightarrow$



Rotation droite





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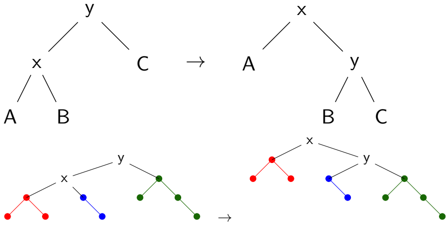

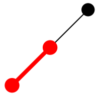

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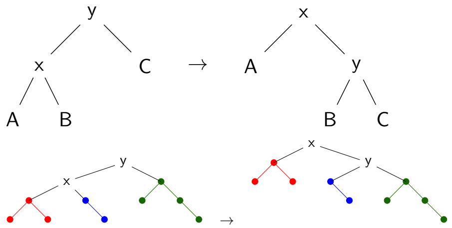
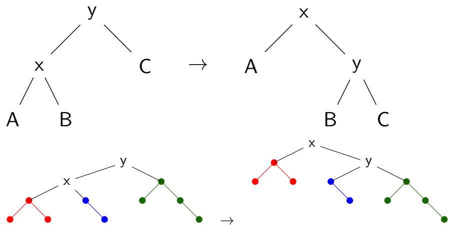
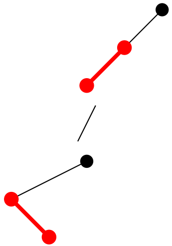
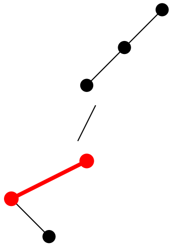


Rotation droite







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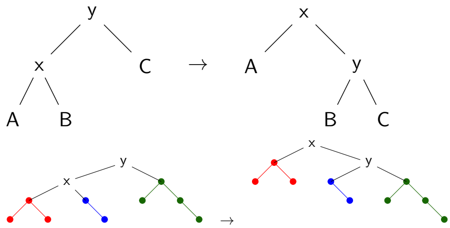

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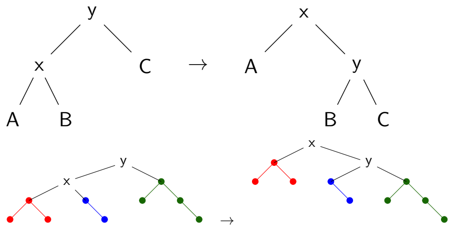

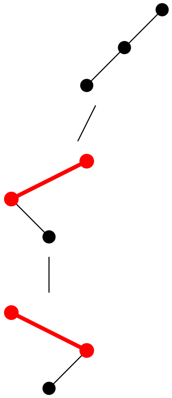

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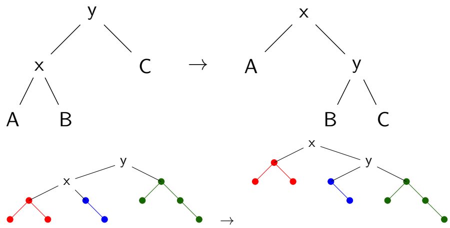
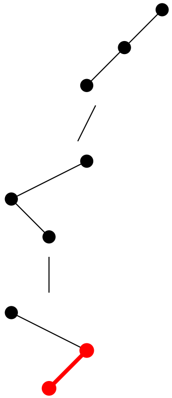
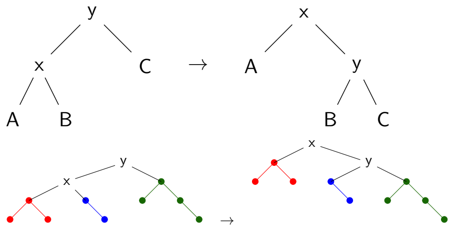
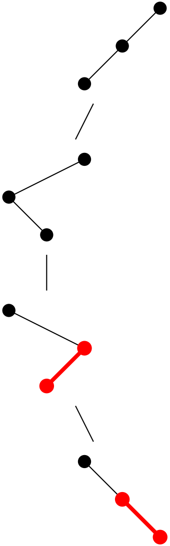




Rotation droite





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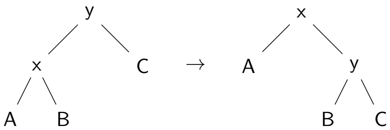
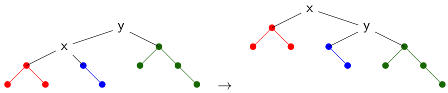
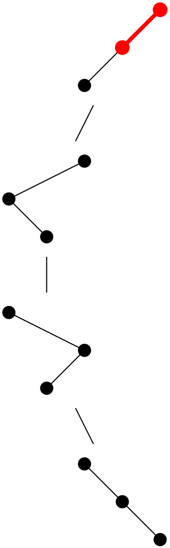
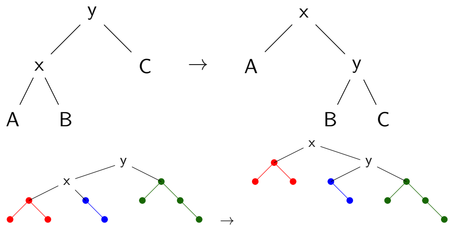
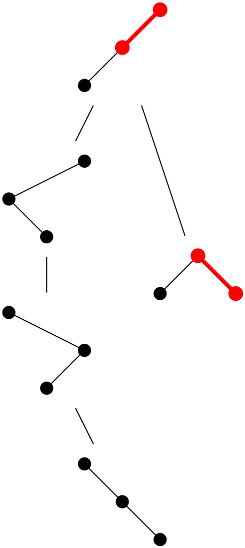




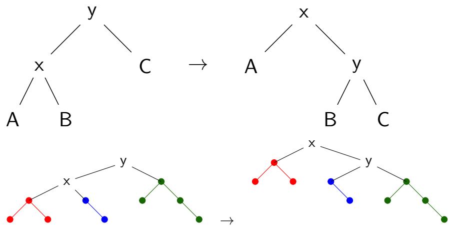


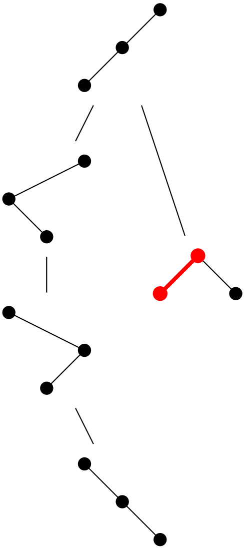
Rotation droite



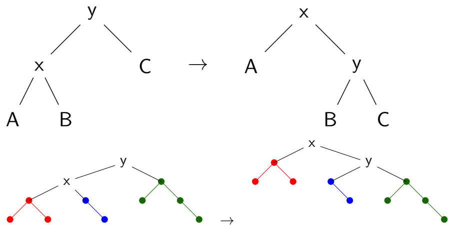


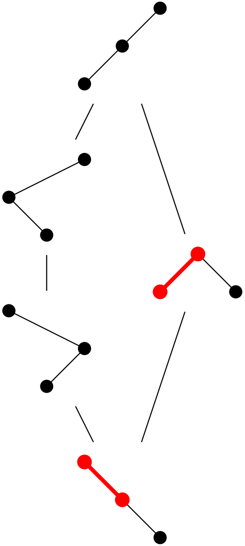
Rotation droite



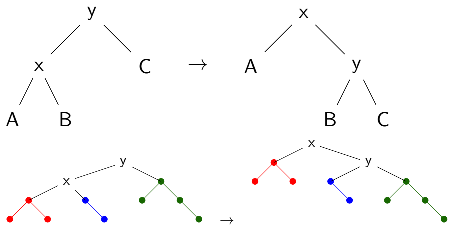


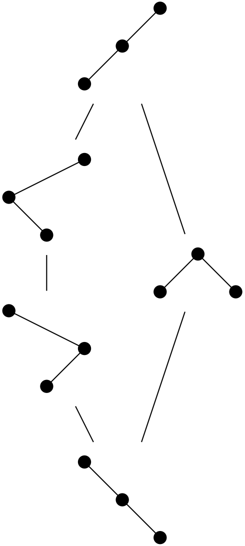
Rotation droite

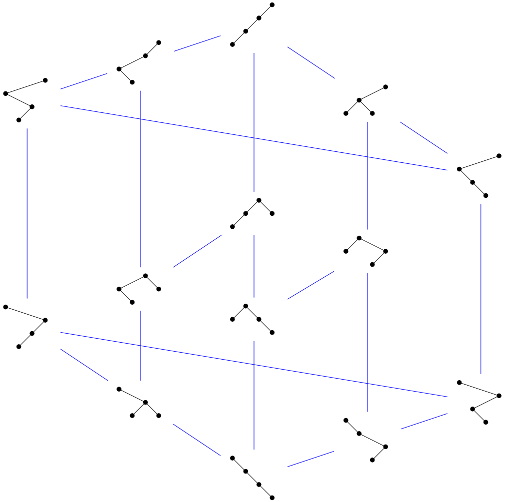




Rotation droite







Posets de m-Tamari

(2011-2012) Bergeron, Préville-Ratelle, *Higher trivariate diagonal harmonics via generalized Tamari posets.*

Posets de m-Tamari

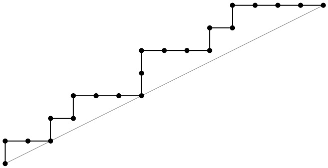
(2011-2012) Bergeron, Préville-Ratelle, *Higher trivariate diagonal harmonics via generalized Tamari posets.*

Structure de treillis

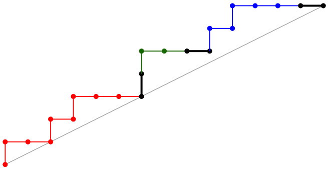
(2011) Bousquet-Mélou, Fusy, Préville-Ratelle, *The number of intervals in the m-Tamari lattices.*

Chemins m-dyck

Exemple $m = 2$.



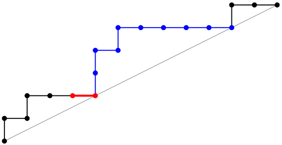
Structure $(m+1)$-aire



Comptés par les nombres de Fuss-Catalan :

$$\frac{1}{mn+1}\binom{(m+1)n}{n}$$

Rotation



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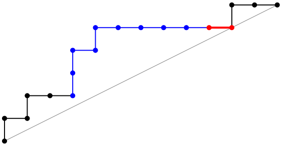
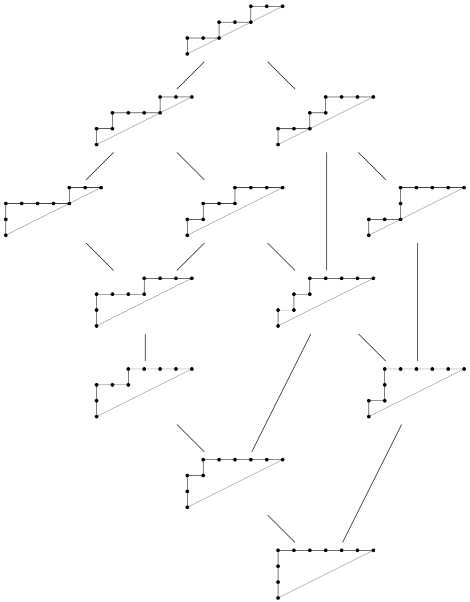
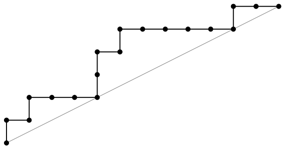




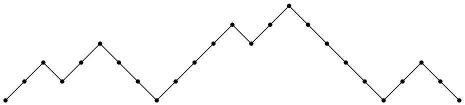
Les posets de m-Tamari sont des treillis

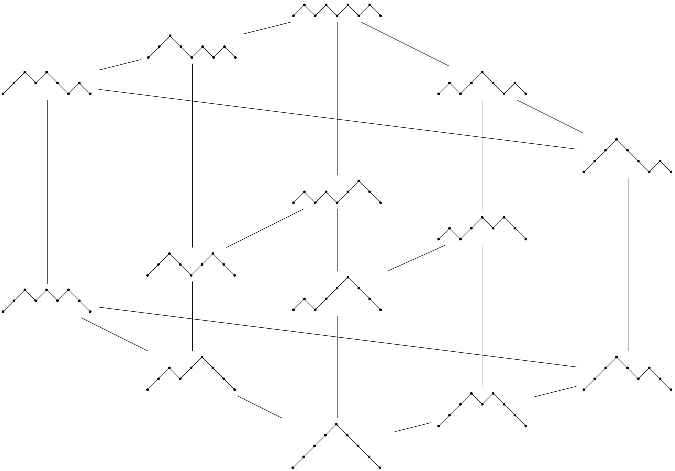
Bousquet-Mélou, Fusy, Préville-Ratelle : plongement des posets m-Tamari dans le treillis de Tamari $n \times m$.

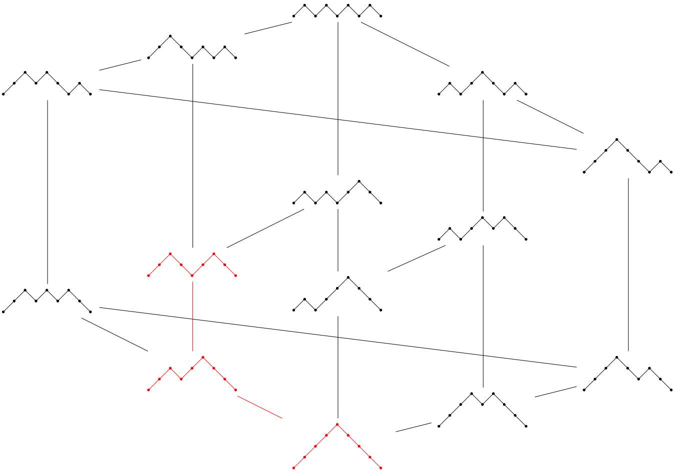
chemin m-Dyck

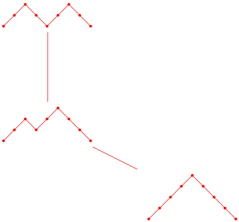


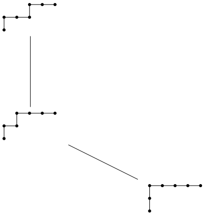
chemin de Dyck



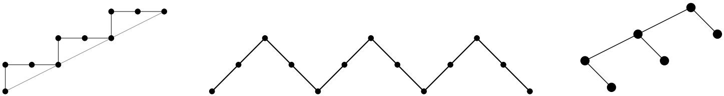


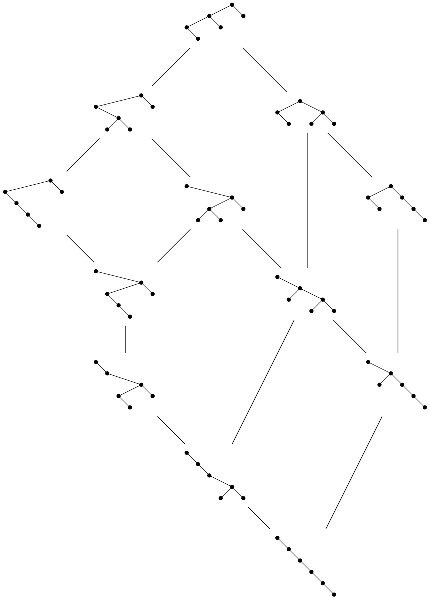


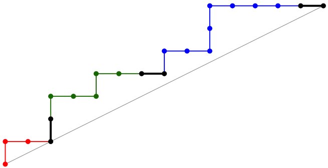


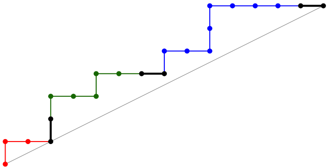


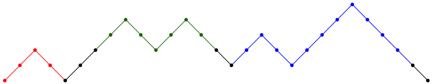
Arbres m-binaires

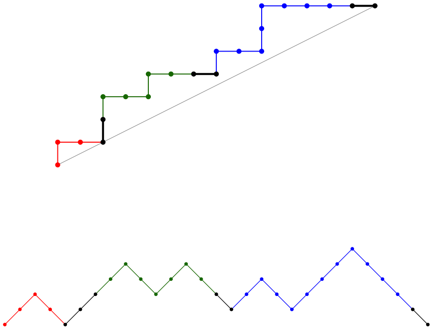


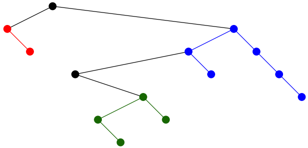




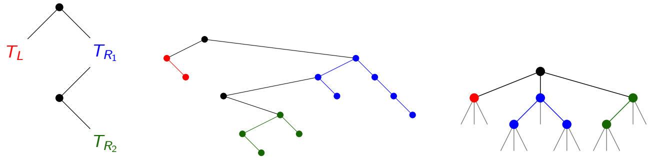




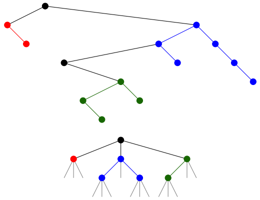




Structure ternaire



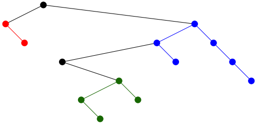
Relation de couverture



$\longrightarrow$

$\longrightarrow$

Relation de couverture



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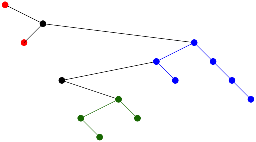

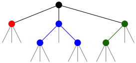

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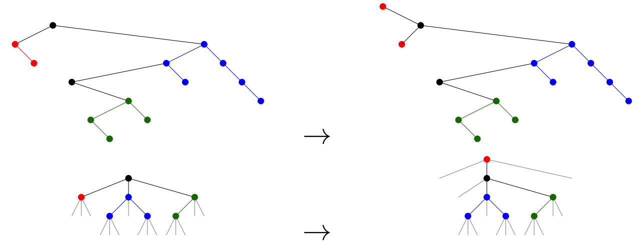
Relation de couverture



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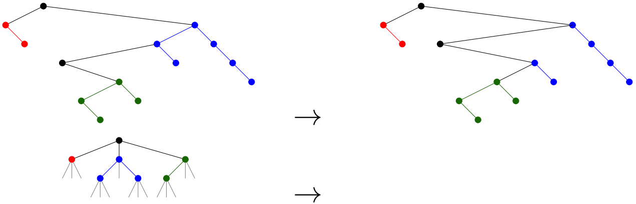

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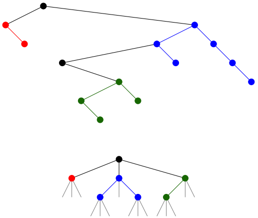

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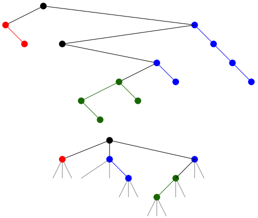
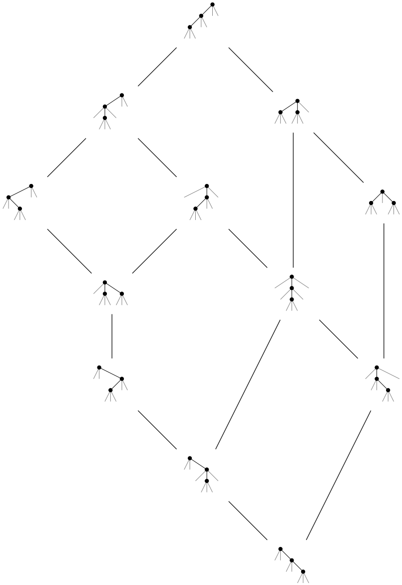




Énumération des intervalles

- 2007, Chapoton, pour Tamari :

$$\frac{2}{n(n+1)}\binom{4n+1}{n-1}$$

- 2011, Bousquet-Mélou, Fusy, Préville-Ratelle, pour m-Tamari :

$$\frac{m+1}{n(mn+1)}\binom{(m+1)^2n+m}{n-1}$$

Équation fonctionnelle sur la série génératrice

(Tamari)

$$\Phi = 1 + \mathrm{B}(\Phi, \Phi)$$

$$\mathrm{B}(f, g) = xf(x)\Delta(g)$$

$$\Delta(g) = \frac{xg(x) - g(1)}{x - 1}$$

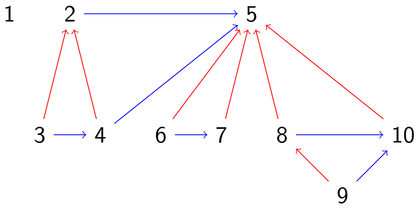
Équation fonctionnelle sur la série génératrice

(m-Tamari)

$$\Phi^{(2)} = 1 + \mathrm{B}^{(2)}(\Phi^{(2)}, \Phi^{(2)}, \Phi^{(2)})$$

$$\mathrm{B}^{(2)}(f, g_1, g_2) = xf(x)\Delta(g_1\Delta(g_2))$$

$$\Delta(g) = \frac{xg(x) - g(1)}{x - 1}$$

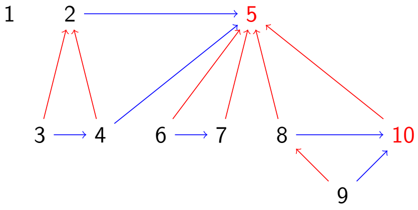


Définition

Un intervalle-poset est un poset de taille n étiqueté par $1, \dots, n$ *tel que*

- *Si* $a < c$ *et* $a \lhd c$ *alors* $b \lhd c$ *pour tout* $a < b < c$.
- *Si* $a < c$ *et* $c \lhd a$ *alors* $b \lhd a$ *pour tout* $a < b < c$.

On note $a \lhd b$ *pour a inférieur à b dans le poset.*

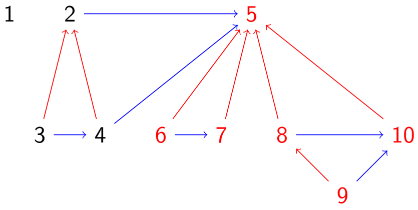


Définition

Un intervalle-poset est un poset de taille n étiqueté par $1, \ldots, n$ tel que

- *Si $a < c$ et $a \lhd c$ alors $b \lhd c$ pour tout $a < b < c$.*
- *Si $a < c$ et $c \lhd a$ alors $b \lhd a$ pour tout $a < b < c$.*

On note $a \lhd b$ pour a inférieur à b dans le poset.



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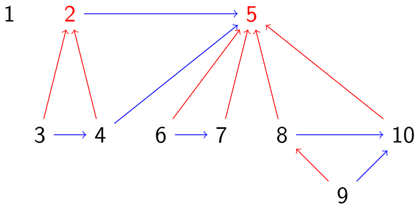
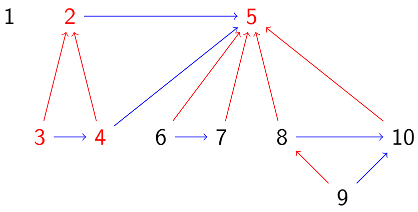


Définition

Un intervalle-poset est un poset de taille n étiqueté par $1, \dots, n$ tel que

- *Si $a < c$ et $a \lhd c$ alors $b \lhd c$ pour tout $a < b < c$.*
- *Si $a < c$ et $c \lhd a$ alors $b \lhd a$ pour tout $a < b < c$.*

On note $a \lhd b$ pour a inférieur à b dans le poset.



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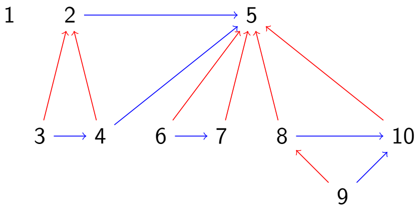

Théorème (Châtel, P.)

Les intervalles-posets sont en bijection avec les intervalles du treillis de Tamari.

Les intervalles de m-Tamari sont un sous-ensemble des intervalles de Tamari.