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# ASCHEA: New Results Using Adaptive Segregational Constraint Handling

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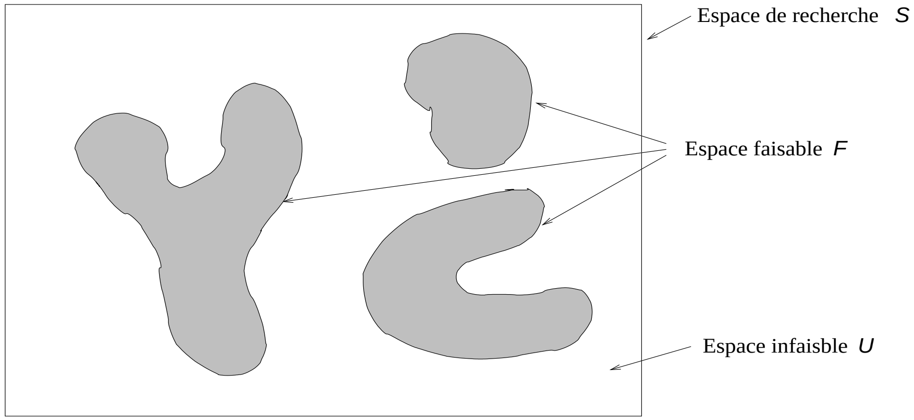
## 1 – Constrained Optimisation Problems

Optimize $f(\vec{x}), \vec{x} = (x_1, \ldots, x_n) \in \mathcal{F} \subseteq \mathcal{S} \subseteq \mathbb{R}^n$,

such that:

$$\begin{cases} g_i(\vec{x}) \leq 0, & \textbf{for } i = 1, \ldots, q & \textbf{inequality constraints} \\ h_j(\vec{x}) = 0, & \textbf{for } j = q+1, \ldots, m & \textbf{equality constraints} \end{cases}$$

where $f$, $g_i$ and $h_j$ are real-valued functions on the search space $\mathcal{S}$. The satisfaction of the set of constraints ($g_i$, $h_j$) defines the feasible region $\mathcal{F}$.



## Penalty methods: penalize infeasible solutions

$$eval(\vec{x}) = \begin{cases} f(\vec{x}), & \textbf{if } \vec{x} \in \mathcal{F} \\ f(\vec{x}) + penal(\vec{x}), & \textbf{otherwise} \end{cases}$$

where $penal(\vec{x})$ is positive for minimizing and negative for maximizing.

$$penal(\vec{x}) = \sum_{j=1}^{q} \alpha_j g_j^+(\vec{x}) + \sum_{j=q+1}^{m} \alpha_j |h_j|(\vec{x}),$$

where $x^+$ denotes the positive part of $x$ and the real numbers $\alpha_j$, $j = 1 \ldots m$ are the *penalty coefficients*.

# 2 – The Adaptive Segregational algorithm: ASCHEA

**Goal: Maintain population diversity**

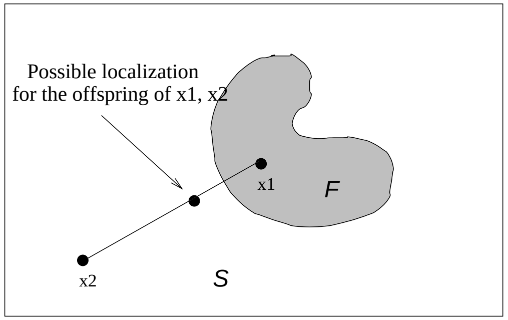
$\Longrightarrow$ maintain both feasible and infeasible individuals in the population.

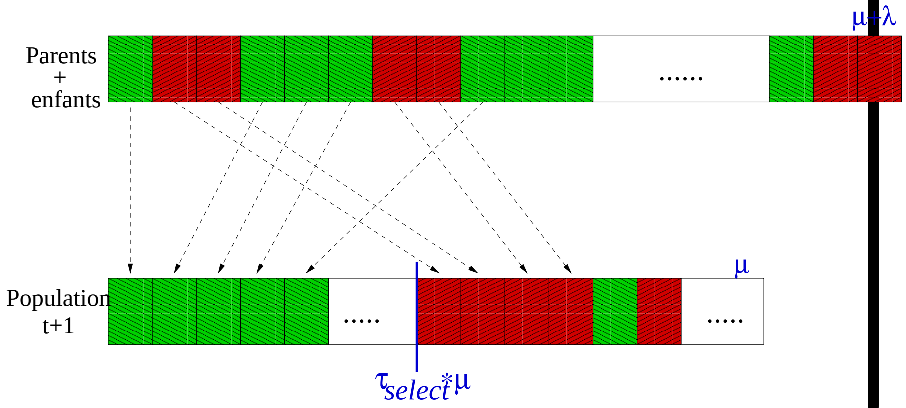
**ASCHEA**

| (fitness computation) | (recombination) | (selection) |
|---|---|---|
| **Adaptive penalty** | **Seduction/Selection** | **Segregated selection** |

If not enough feasibles
penalty coeff ↗

If not enough infeasibles
penalty coeff ↘



## 3 – ASCHEA+: Improving the Adaptive Penalty Function

ASCHEA: a single penalty cœfficient $\alpha(t)$

$\Longrightarrow$ $\alpha(t)$ adapted according to the proportion $\tau_t$ of feasible individuals in the population

$$\begin{array}{ll} \textbf{if } (\tau_t > \tau_{target}) & \alpha(t+1)(\vec{x}) = \alpha(t)/fact \\ \textbf{otherwise} & \alpha(t+1)(\vec{x}) = \alpha(t) * fact \end{array}$$

$\tau_{target}$ and $fact > 1$: user-defined parameters (typically, $\tau_{target} \approx 0.5$)

ASCHEA+: Extension to multiple penalty coefficient

$$\boxed{penal(\vec{x}) = \sum_{j=1}^{q} \alpha_j(t) g_j^+(\vec{x}) + \sum_{j=q+1}^{m} \alpha_j(t)|h_j|(\vec{x}),}$$

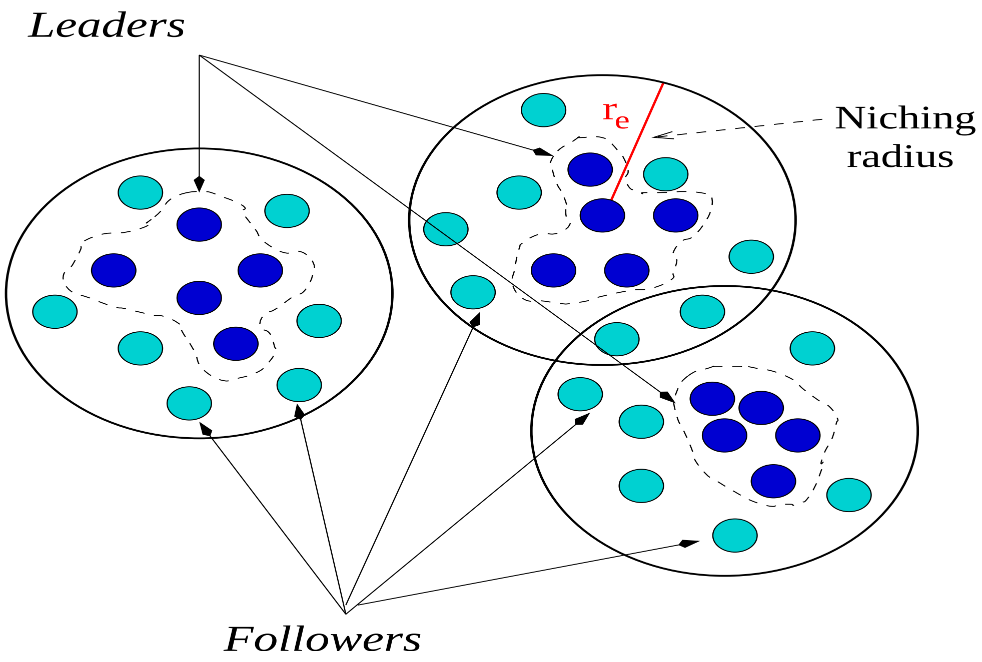
Adapt each $\alpha_i$ independently using the same strategy:

$$\boxed{\begin{array}{ll} \textbf{si}\ (\tau_t(j) > \tau_{target}) & \alpha_j(t+1)(\vec{x}) = \alpha_j(t)/fact \\ \textbf{else} & \alpha_j(t+1)(\vec{x}) = \alpha_j(t) * fact \end{array}}$$

$\tau_t(i)$ is the proportion of individuals feasible for constraint $i$

## 4 – ASCHEA+: Adding Niching

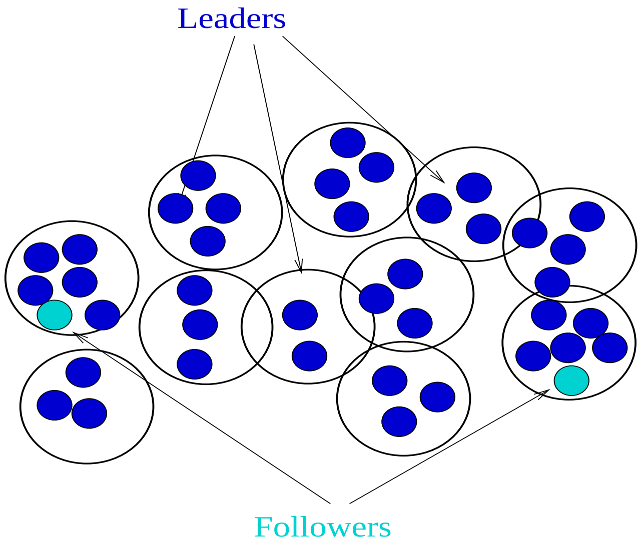
Niching method: Elitist clearing (*Pétrowski, 96*) adapted to ASCHEA

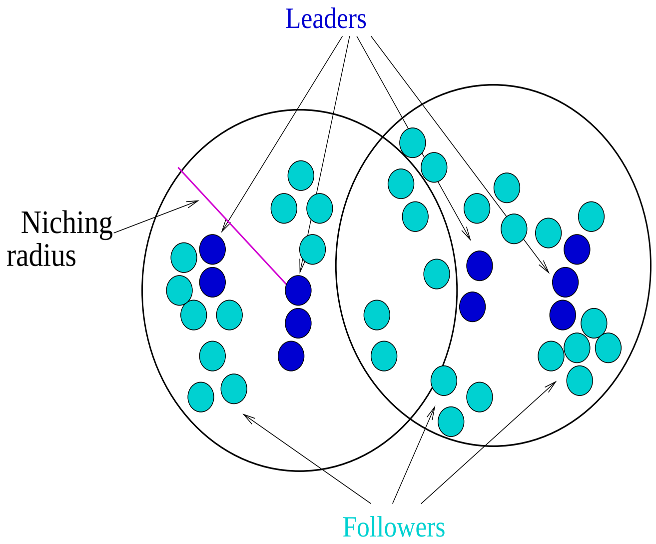



1. Classify individuals on *leaders* and *followers*.
2. Apply segregational selection to the *leaders*.
3. If necessary , complete with *followers*.

## 4.1 – Adaptive niching radius

Goal: adapt the niching radius (r) according to the proportion of *leaders* ($\tau_{leaders}$) and *followers* ($\tau_{followers}$) in the population.

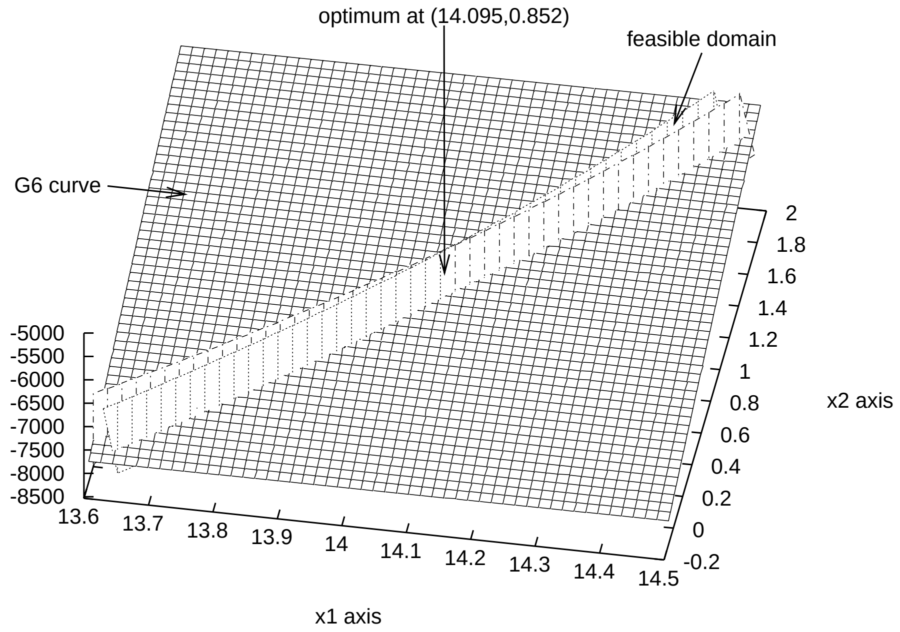



| | |
|---|---|
| if $\tau_{leaders} > \tau_{clear}$ | $r(t+1) = r(t)/fact$ |
| else if $\tau_{followers} > \tau_{clear}$ | $r(t+1) = r(t) * fact$ |

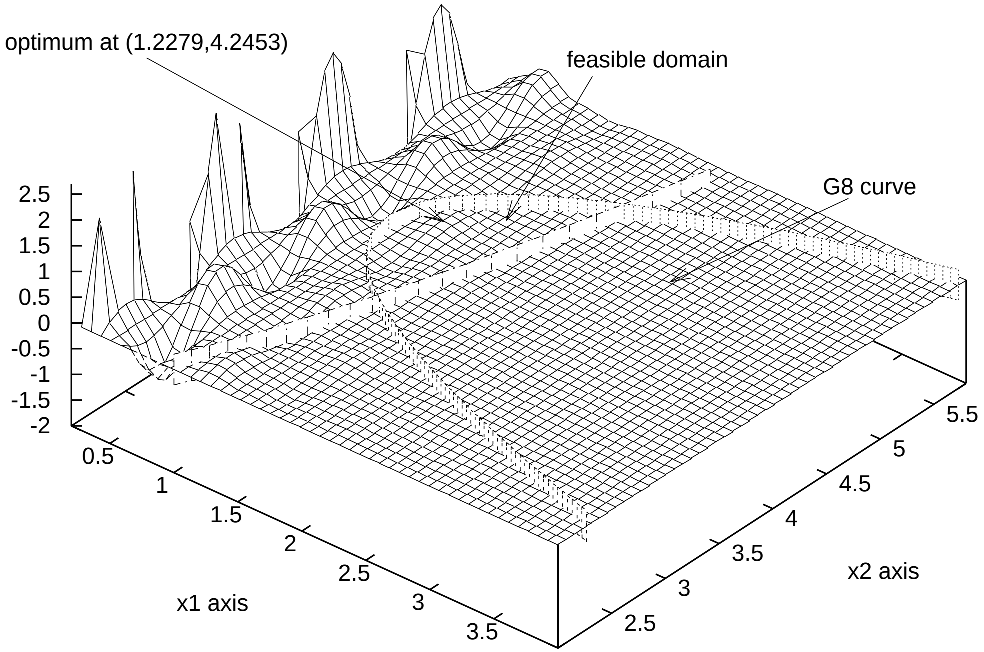
## 5 – ASCHEA+: Experimental Behavior

**Test 1 (G6)** $\Longrightarrow$ optimum lies on the boundary of $\mathcal{F}$




**G6 landscape**. *The constraint boundaries have been artificially made visible.*

**Test 2 (G8)** $\Longrightarrow$ two quasi-optima inside $\mathcal{F}$




**G8 landscape**. *Note that function G8 has huge peaks toward the $x_2$ axis that are not shown here.*

## 5.1 – Experimental conditions

| | |
|---|---|
| **Evol. engine**: | (100+300)-ES segregational selection strategy |
| **Mutation**: | Gaussian mutation with self-adaptive standard deviations |
| | $\sigma_0 = 0.03$, $\tau = 0.1$, $\tau' = 1$, $p_m = 0.9$ |
| **Recombination**: | Arithmetical crossover, $p_c = 0.9$ |
| **ASCHEA**: | $\tau_{select} = 0.3$, $\tau_{target} = 0.5$, $fact = 1.1$ |
| **Niching**: | $\tau_{clear} = 0.4$, $niche\_capacity = 8$ |
| **Max generations**: | 5000 |
| **# runs**: | 31 |

# 6 – Extensive experimental results

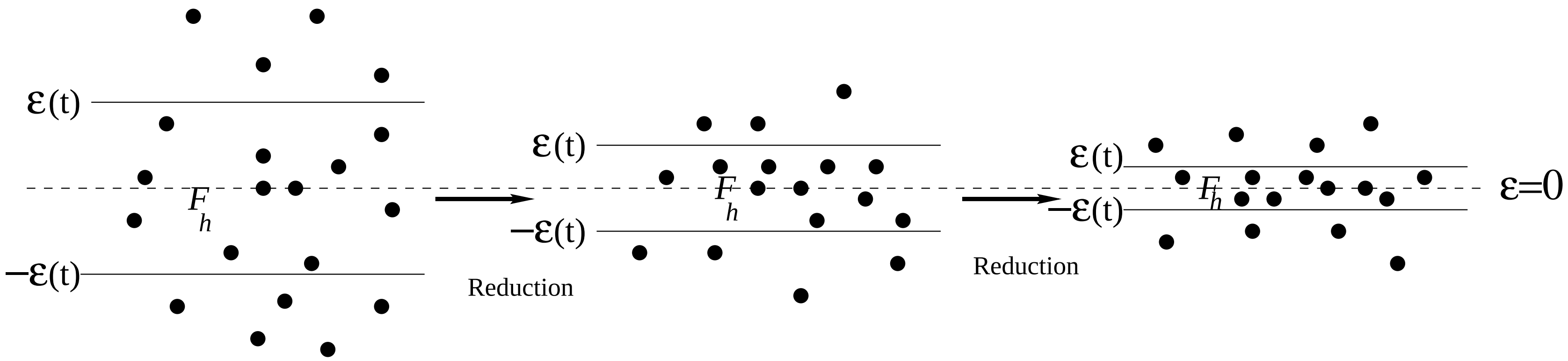
| Function | Opt. value | Opt. found | | | After 5000 gen | | |
|---|---|---|---|---|---|---|---|
| | | Hits | best (nb gen) | Avg. (nb gen) | best | median | Avg. |
| G1 | -15 | 29 | 262 | 330 | **-15** | -15 | -14.84 |
| G2+ | 0.803619 | – | – | – | **0.803614** | 0.794568 | 0.88949 |
| G3* | 1.0 | 1 | 4712 | 4712 | **1** | 0.999998 | 0.99989 |
| G4 | -30665.5 | **31** | 95 | 217 | **-30665.5** | -30665.5 | -30665.5 |
| G5*+ | 5126.4981 | 30 | 98 | 127 | **5126.5** | 5126.5 | 5126.53 |
| G6 | -6961.81 | **31** | 92 | 150 | **-6961.81** | -6961.81 | -6961.81 |
| G7 | 24.3062 | – | – | – | 24.3323 | 24.6162 | 24.6636 |
| G8 | 0.095825 | **31** | 12 | 32 | **0.095825** | 0.095825 | 0.095825 |
| G9 | 680.630 | 1 | 4567 | 4567 | **680.630** | 680.635 | 680.641 |
| G10+ | 7049.33 | – | – | – | **7049.42** | 7272.19 | 7615.2 |
| G11* | 0.75 | **31** | 27 | 37 | **0.75** | 0.75 | 0.75 |

Summary of results on 11 test cases (from G1 to G11). The stars * denote that each equality constraint has been turned into one inequality constraint. The '+' denote that ASCHEA+ was applied with niching. ( G2$\rightarrow n = 20$,G3$\rightarrow n = 10$).

# 7 – Handling Equality Constraints

**Idea**: Start with a large feasible domain

reduce it gradually along evolution




**Two approaches**: dynamic vs adaptive adjustment

## 7.1 – Dynamic Adjustment

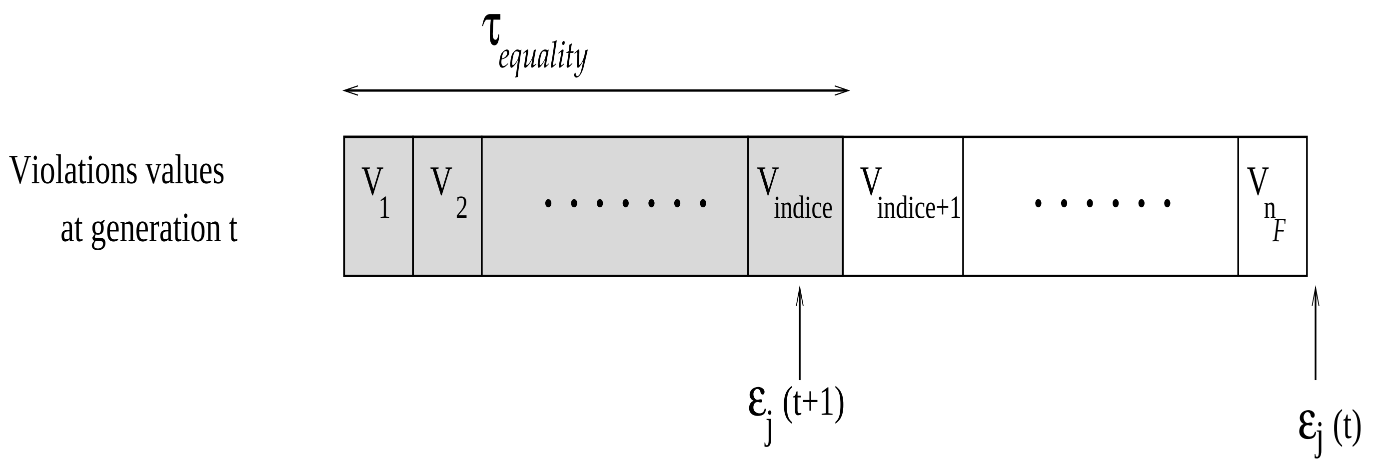
$$\begin{array}{ll} \textbf{if } (\tau_t(j) > \tau_{reduct}) & \epsilon_j(t+1) = \epsilon_j(t)/fact_\epsilon \\ \textbf{otherwise} & \epsilon_j(t+1) = \epsilon_j(t) \end{array}$$

where $\tau_{reduct}$ and $fact_\epsilon > 1$ are user-defined parameters

$$\tau_{reduct} > \tau_{target}$$

**But** all feasible individuals might become infeasible!

## 7.2 – Adaptive Adjustment




The $\epsilon_j$ value is updated such as a proportion $\tau_{equality}$ of the population (individuals having the $V_1, V_2, \cdots, V_{indice}$ violations) still feasible.

**But** $\epsilon_j(t)$ might stay away from 0

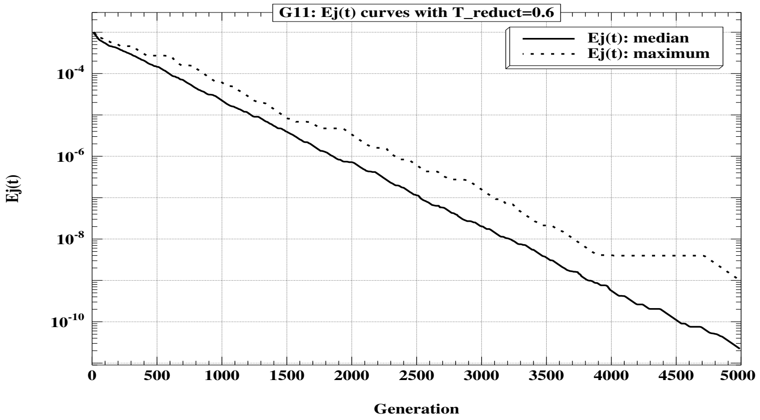
# 8 – ASCHEA++: Experimental Behavior

Three test-cases: G3, G11 (1 equality constraint), G5 (3 equality constraints).

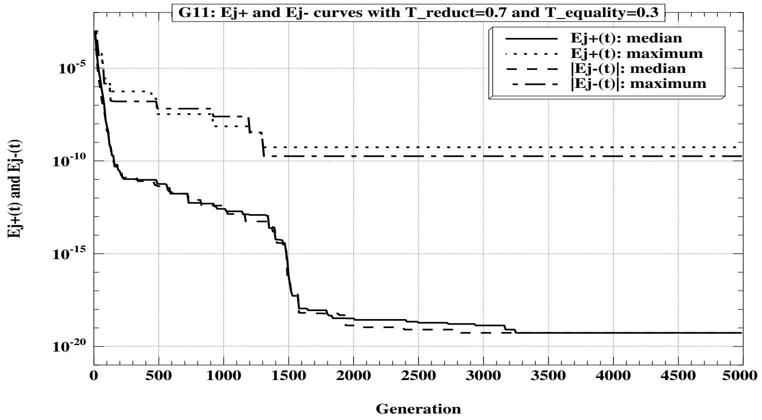
Parameters:

- dynamic adjustment: $fact_{\epsilon}$=1.01, $\tau_{reduct} = 0.6$,

- adaptive adjustment: $\tau_{reduct} = 0.7$, $\tau_{equality} = 0.3$




(a): Dynamic adjustment




(b): Adaptive adjustment

G11: Evolution of median and maximum $\epsilon_j$ (dynamic adjustment), $\epsilon_j^+$ and $\epsilon_j^-$ (adaptive adjustment) values.

## 8.1 – Experimental results

| Approach | Case | Fitness | | | Violation | | |
|---|---|---|---|---|---|---|---|
| | | Best | Med. | Avg. | Best | Med. | Avg. |
| Dynamic $\tau_{reduct}$ 0.6 | G3 | 1 | 0.999993 | 0.9932 | $9.18^{-9}$ | $1.98.10^{-7}$ | $4.37.10^{-5}$ |
| | G5 | 5126.52 | 5154.13 | 5168.9 | $1.6.10^{-8}$ | $6.1.10^{-7}$ | $2.6.10^{-4}$ |
| | G11 | 0.75 | 0.75 | 0.7519 | $3.1.10^{-12}$ | $2.1.10^{-11}$ | $7.4.10^{-11}$ |
| Adaptive $\tau_{reduct}$ 0.7 $\tau_{equality}$ 0.3 | G3 | 0.99919 | 0.9335 | 0.8925 | 0 | $2.2.10^{-16}$ | $1.8.10^{-16}$ |
| | G5 | 5126.53 | 5159.19 | 5159.24 | $1.1.10^{-6}$ | $2.2.10^{-4}$ | $4.5.10^{-3}$ |
| | G11 | 0.75 | 0.75 | 0.7525 | 0 | $5.4.10^{-20}$ | $1.8.10^{-11}$ |

*Results on G3, G5 and G11 – dynamic and adaptive approaches.*

# 9 – Conclusion

| | | |
|---|---|---|
| **ASCHEA+** | $\rightarrow$ | extended penalty function, |
| | $\rightarrow$ | niching method with adaptive radius, |
| | $\Rightarrow$ | improved robustness and better results. |
| **ASCHEA++** | $\rightarrow$ | handling equality constraints with progressive reduction of the feasible space |
| | $\rightarrow$ | two strategies: adaptive and dynamic adjustment |
| | $\Rightarrow$ | satisfactory results<br>need improvements to limit the risk of loss of feasible individuals. |