M1 – Apprentissage

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LRI – LSV

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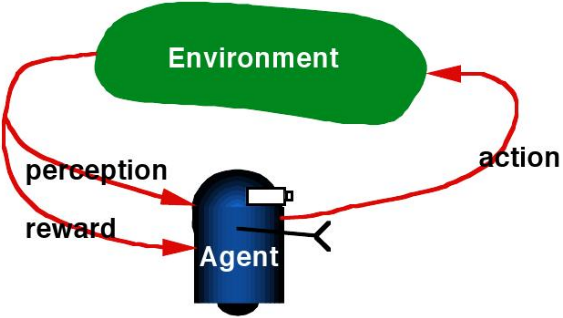
Overview

Reinforcement Learning



Generalities

- An agent, spatially and temporally situated
- Stochastic and uncertain environment
- Goal: select an action in each time step,
- ... in order maximize expected cumulative reward over a time horizon

What is learned ?

A policy = strategy = $\{$ state $\mapsto$ action $\}$

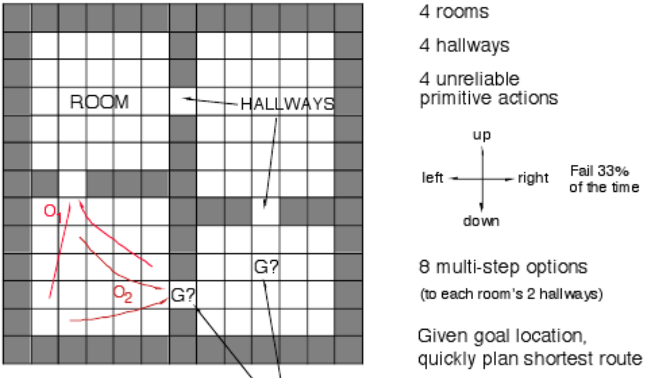
Reinforcement Learning

Context

An unknown world.

Some actions, in some states, bear rewards with some delay [with some probability]

Goal : **find policy (state $\rightarrow$ action) maximizing the expected reward**



Reinforcement Learning, example

World You are in state 34.
Your immediate reward is 3. You have 3 actions

Robot I'll take action 2

World You are in state 77
Your immediate reward is -7. You have 2 actions

Robot I'll take action 1

World You are in state 34 (again)

Markov Decision Property: actions/rewards only depend on the current state.

Reinforcement Learning

Of several responses made to the same situation, those which are accompanied or closely followed by satisfaction to the animal will – others things being equal – be more firmly connected with the situation, so that when it recurs, they will more likely to recur;

those which are accompanied or closely followed by discomfort to the animal will – others things being equal – have their connection with the situation weakened, so that when it recurs, they will less likely to recur;

the greater the satisfaction or discomfort, the greater the strengthening or weakening of the link.
Thorndike, 1911.

Formal background

Notations

- State space $\mathcal{S}$
- Action space $\mathcal{A}$
- Transition model $p(s, a, s') \mapsto [0, 1]$
- Reward $r(s)$

Goal

- Find policy $\pi : \mathcal{S} \mapsto \mathcal{A}$

$$\text{Maximize } E[\pi] = \text{ Expected cumulative reward}$$

(detail later)

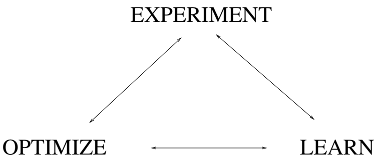
Applications

- Robotics
  Navigation, football, walk,
- Control
  Helicopter, elevators, telecom, smart grids, manufacturing, ...
- Operation research
  Transport, scheduling, ...
- Games
  Backgammon, Othello, Tetris, Go, ...
- Other
  Computer Human Interfaces, ML (Feature Selection, Active learning, Natural Language Processing,...)

Position of the problem

3 interleaved tasks

- Learn a world model (p, r)
- Decide/select (the best) action
- Explore the world



Sources

- Sutton & Barto, Reinforcement Learning, MIT Press, 1998
- `http://www.eecs.umich.edu/~baveja/NIPS05RLTutorial/`

Particular case

If the transition model is known

Reinforcement learning $\rightarrow$ Optimal control

What's hard

Curse of dimensionality

- State: features *size*, *texture*, *color*, ...
  $|\mathcal{S}|$ exponential wrt number of features
- Not all features are always relevant

Example:

see	swann	white	—
	swann	black	take a video
	bear	—	flee

What's hard

Curse of dimensionality

- State: features *size*, *texture*, *color*, … …
  $|\mathcal{S}|$ exponential wrt number of features
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Example:

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	bear	—	flee

Time horizon – Bounded rationality

- T.h. is infinite: eternity. NEVER
- Finite, unknown: reach the goal asap
- Finite: reach the goal in T time steps
- Bounded rationality: find as fast as possible a decent policy (finding an approximation of the goal).

Overview

Formalisation

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- State space $\mathcal{S}$
- Action space $\mathcal{A}$
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  - deterministic: $s' = t(s, a)$
  - probabilistic: $p(s, a, s') \in [0, 1]$.
- Reward $r(s)$ **bounded**
- Time horizon H (finite or infinite)

Goal

- Find policy (strategy) $\pi : \mathcal{S} \mapsto \mathcal{A}$
- which maximizes (discounted) cumulative reward from now to timestep H

$$\sum_t r(s_t)$$

Formalisation

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$$\sum_{t=1}^{H} \gamma^t r(s_t) \qquad \gamma < 1$$

Formalisation

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$$\mathbb{E}_{s_0,\pi}[\sum_{t=1}^{\infty} \gamma^t r(s_t)]$$

Markov Decision Process

But can we define $P^a_{ss'}$ and $r(s)$?

- YES, if all necessary information is in s
- NO, otherwise
  - If state is partially observable

Goal: arrive in the third branch

  - If environment (reward and transition distribution) is changing
    Reward for *first* photo of an object by the satellite

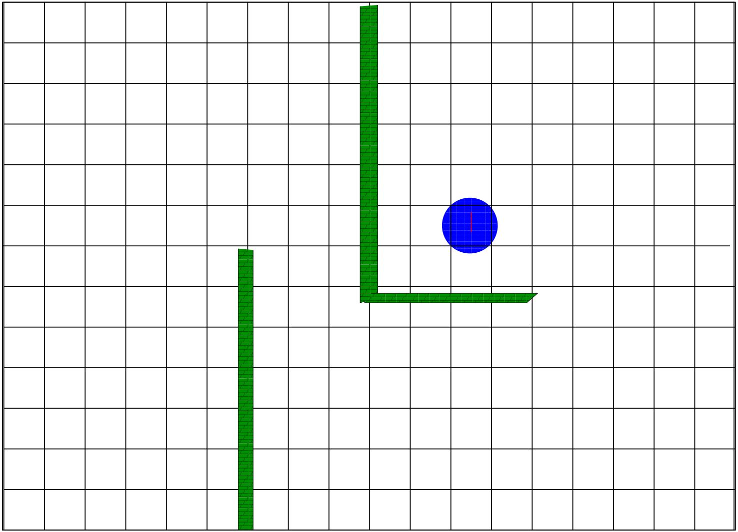
The Markov assumption

$$P(s_{h+1}|s_0\ a_0\ s_1\ a_1 \ldots s_h\ a_h) = P(s_{h+1}|s_h\ a_h)$$

Everything you need to know is the current (state, action).

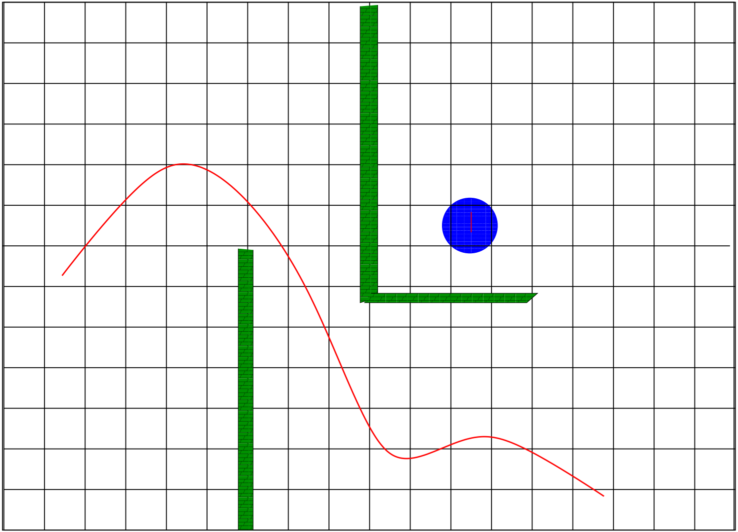
Find the treasure

Single reward: on the treasure.

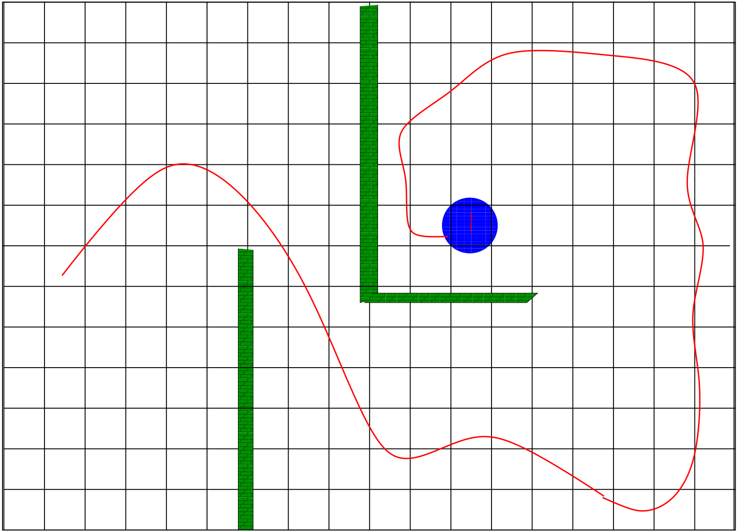


Wandering robot

Nothing happens...

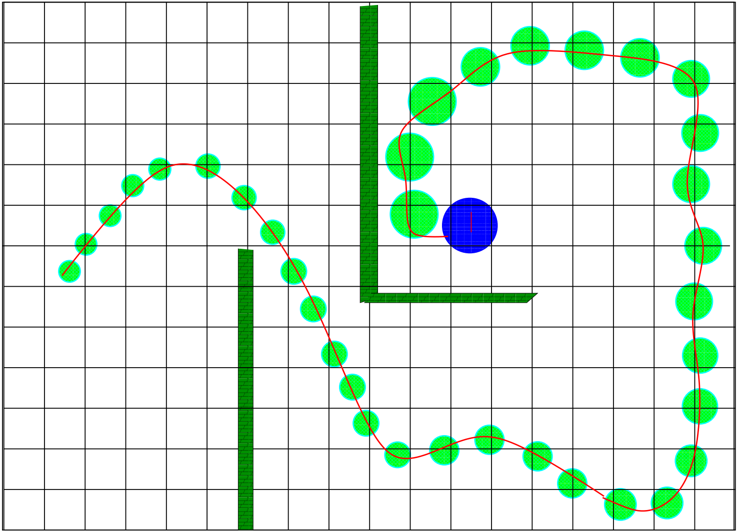


The robot finds it



Robot updates its value function

$V(s, a) ==$ "distance" to the treasure *on the trajectory*.

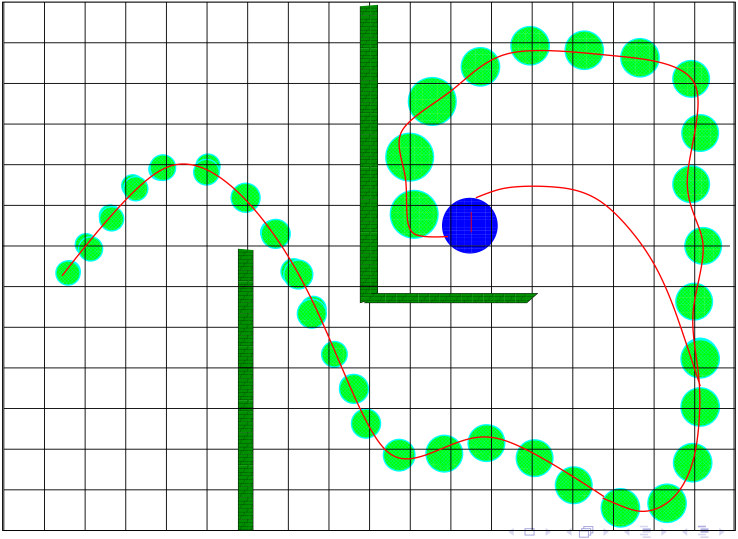


Reinforcement learning

* Robot most often selects $a = \arg\max V(s,a)$
* and sometimes explores (selects another action).

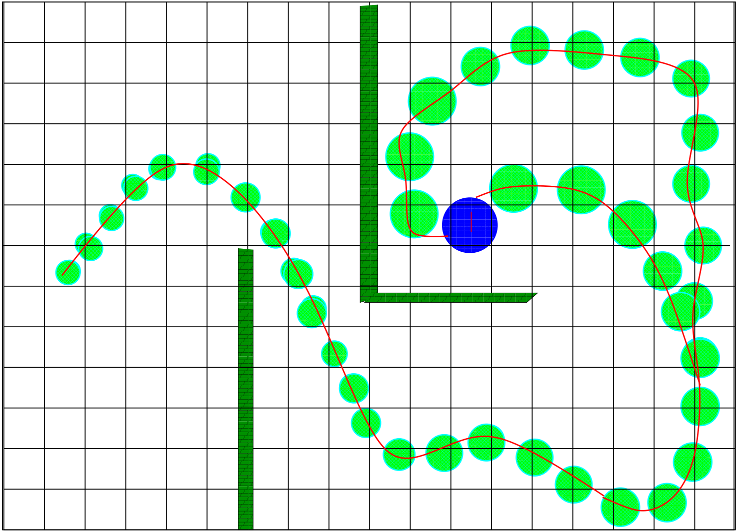
Reinforcement learning

* Robot most often selects $a = \arg\max V(s,a)$
* and sometimes explores (selects another action).
* Lucky exploration: finds the treasure again



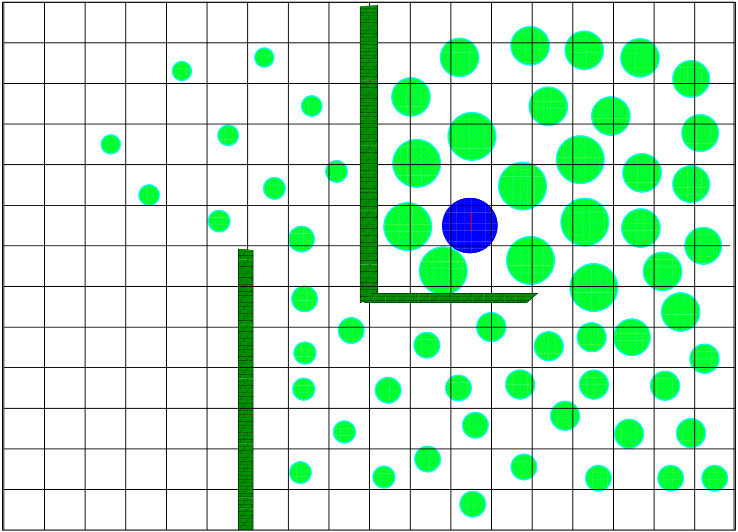
Updates the value function

* Value function tells how far you are from the treasure *given the known trajectories*.



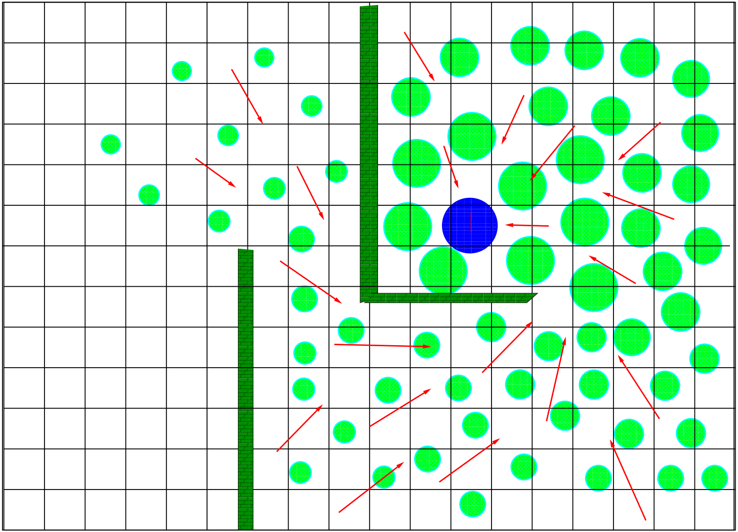
Finally

* Value function tells how far you are from the treasure



Finally

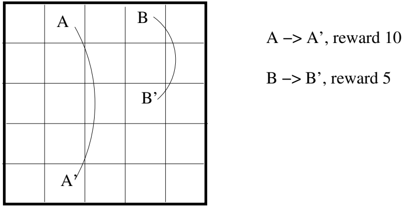
Let's be greedy: selects the action maximizing the value function



Exercize

Uniform policy

- States: squares
- Actions: north, south, east, west.
- Rewards: -1 if you would get outside; 10 in A; 5 in B
- Transition model: as expected (South, North etc, except: in A, any action sends you in A'; in B any action sends you in B'.



Compute the value function

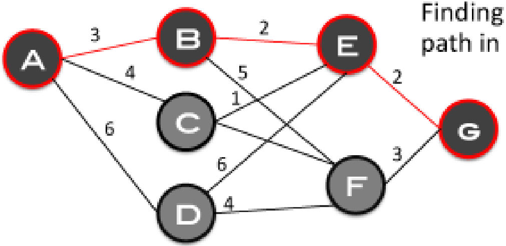
Underlying: Dynamic programming

Principle

- Recursively decompose the problem in subproblems
- Solve and propagate

An example

$$\ell(\text{shortest path } (A,B)) \leq \ell(sp(A,C)) + \ell(sp(C,B))$$



Finding the shortest path in a graph

Approaches

- Value function
  - Value iteration
  - Policy iteration
- Temporal differences
- Q-learning
- Direct policy search
  optimization in the π space

Stochastic optimization

Policy and value function 1/3

Finite horizon, deterministic transition

$$V_\pi(s_0) = r(s_0) + \sum_{h=1}^{H} r(s_h)$$

where $s_{h+1} = t(s_h, a_h = \pi(s_h))$

Policy and value function 1/3

Finite horizon, deterministic transition

$$V_\pi(s_0) = r(s_0) + \sum_{h=1}^{H} r(s_h)$$

where $s_{h+1} = t(s_h, a_h = \pi(s_h))$

Finite horizon, stochastic transition

$$V_\pi(s_0) = r(s_0) + \sum_{h=1}^{H} \mathbf{p(s_{h-1}, a_{h-1} = \pi(s_{h-1}), s_h)} r(s_h)$$

where $s_{h+1} = s$ with proba $p(s_h, a_h = \pi(s_h), s)$

Policy and value function, 2/3

Finite horizon, stochastic transition

$$V_\pi(s_0) = r(s_0) + \sum_{h=1}^{H} \mathbf{p}(\mathbf{s_{h-1}}, \mathbf{a_{h-1}} = \pi(\mathbf{s_{h-1}}), \mathbf{s_h}) r(s_h)$$

where $s_{h+1} = s$ with proba $p(s_h, a_h = \pi(s_h), s)$

Infinite horizon, stochastic transition

$$V_\pi(s_0) = r(s_0) + \sum_{h=1}^{H} \gamma^{\mathbf{h}} \mathbf{p}(\mathbf{s_{h-1}}, \mathbf{a_{h-1}} = \pi(\mathbf{s_{h-1}}), \mathbf{s_h}) r(s_h)$$

with discount factor γ, $0 < \gamma < 1$

Remark

$\gamma < 1 \rightarrow V < \infty$

γ small $\rightarrow$ myopic agent.

Value function and Q-value function

Value function

$$V : S \mapsto \mathbb{R}$$

$V_\pi(s)$: utility of state s when following policy π

Improving π by using V_π requires to know the transition model:

$$\pi(s) \to \text{ arg max }_a p(s, a, s') V_\pi(s')$$

Q function

$$Q : (S \times A) \mapsto \mathbb{R}$$

$Q_\pi(s, a)$: utility of selecting action a in state s when following policy π

Improving π by using Q_π is straightforward:

$$\pi(s) \to \text{ arg max }_a Q_\pi(s, a)$$

Optimal policies

From value function to a better policy

$$\pi(s) = argmax_a\{p(s, a, s')V_\pi(s')\}$$

From policies to optimal value function

$$V^*(s) = max_\pi V_\pi(s)$$

From value function to optimal policy

$$\pi^*(s) = argmax_a\{p(s, a, s')V^*(s')\}$$

Linear and dynamic programming

If transition model and reward function are known

Step 1

$$\pi(s) := \arg\max_a \left\{ \sum_{s'} p(s,a,s') \left(r(s') + \gamma V(s') \right) \right\}$$

Step 2

$$V(s) := \sum_{s'} p(s, a = \pi(s), s') \left(r(s') + \gamma V(s') \right)$$

Properties

Converges eventually toward the optimum if all states, actions are considered.

Value iteration

Bellman equation

Iterate

$$V_{k+1}(s) := \max_a \left\{ \sum_{s'} p(s,a,s') \left(r(s') + \gamma V_k(s') \right) \right\}$$

Stop when

$$max_s |V_{k+1}(s) - V_k(s)| < \epsilon$$

Initialisation

- arbitrary
- educated is better

see Inverse Reinforcement Learning

Policy iteration

Principle

- Modify π step 1
- Update V until convergence step 2

Getting faster

- Don't wait until V has converged before modifying π.

Discussion

Policy and value iteration

- Must wait until the end of the episode
- Episodes might be long

Can we update V on the fly ?

- I have estimates of how long it takes to go to RER, to catch the train, to arrive at Cité-U
- Something happens on the way (bump into a friend, chat, delay, miss the train,...)
- I can update my estimates of when I'll be home...

TD(0)

1. Initialize V and π
2. Loop on episode
   2.1 Initialize s
   2.2 Repeat

$$\begin{array}{l} \text{Select action } a = \pi(s) \\ \text{Observe } s' \text{ and reward } r \\ V(s) \leftarrow V(s) + \alpha(\underbrace{r + \gamma V(s')}_{R} - V(s)) \\ s \leftarrow s' \end{array}$$

   2.3 Until s' terminal state

Discussion

Update on the spot ?

- Might be brittle
- Instead one can consider several steps

$$R = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2})$$

Find an intermediate between

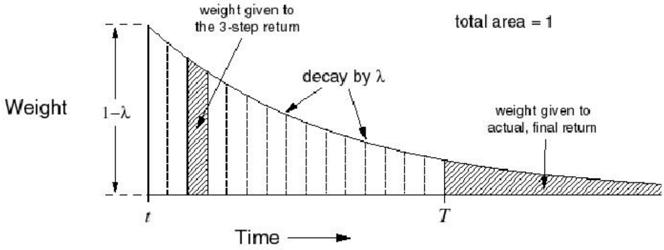
- Policy iteration

$$R_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \dots$$

- TD(0)

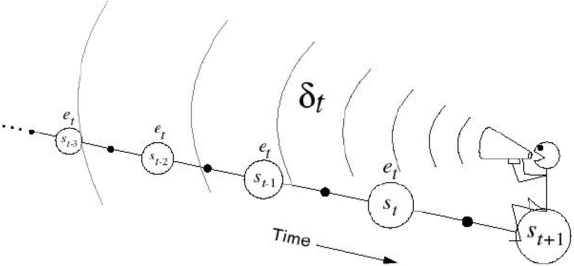
$$R_t = r_{t+1} + \gamma V_t(s_{t+1})$$

TD(λ), intuition



$$R_t^\lambda = \underbrace{(1-\lambda)\sum_{n=1}^{T-t-1}\lambda^{n-1}R_t^{(n)}}_{} + \underbrace{\lambda^{T-t-1}R_t}_{}$$

TD(λ), intuition, followed



$$\delta_t = r_{t+1} + \gamma V_t(s_{t+1}) - V_t(s_t)$$

TD(λ)

1. Initialize V and π
2. Loop on episode
   2.1 Initialize s
   2.2 Repeat

$$
\begin{array}{l}
a = \pi(s) \\
\text{Observe } s' \text{ and reward } r \\
\delta \leftarrow r + V(s') - V(s) \\
e(s) \leftarrow e(s) + 1 \\
\qquad\qquad \text{For all } s“ \\
\qquad\qquad V(s'') \leftarrow V(s“) + \alpha\delta e(s'') \\
\qquad\qquad e(s'') \leftarrow \gamma\lambda e(s'') \\
s \leftarrow s'
\end{array}
$$

   2.3 Until s' terminal state

Q-learning

Principle: Iterate

- During an episode (from initial state until reaching a final state)
- At some point explore and choose another action;
- If it improves, update $Q(s, a)$:

$$Q(s_t, a_t) \leftarrow \underbrace{Q(s_t, a_t)}_{\text{old value}} + \underbrace{\alpha}_{\text{learning rate}} \times \left[\overbrace{\underbrace{r(s_{t+1})}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \underbrace{\max_{a_{t+1}} Q(s_{t+1}, a_{t+1})}_{\text{max future value}}}^{\text{learned value}} - \underbrace{Q(s_t, a_t)}_{\text{old value}} \right]$$

Equivalent to

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t)(1 - \alpha) + \alpha[r(s_{t+1}) + \gamma \max_{a_{t+1}} Q(s_{t+1}, a_{t+1})]$$

Partial summary

Notations

- State space $\mathcal{S}$
- Action space $\mathcal{A}$
- Transition model
  - deterministic: $s' = t(s, a)$
  - probabilistic: $p(s, a, s') \in [0, 1]$.
- Reward $r(s)$ **bounded**
- Time horizon H (finite or infinite)

Policy $\pi \leftrightarrow$ Value function $V(s)$ (or $Q(s, a)$)

1 Update V Iterate [until convergence]

2 Modify π

Reinforcement Learning, 2

Strengths

- Optimality guarantees (converge to global optimum)...

Weaknesses

- ...if each state is visited often, and each action is tried in each state
- Number of states: exponential wrt number of features

Behavioral cloning

Sammut, Bain 95

Input

- Traces (s_t, a_t) of expert

Supervised learning

- Learn $\hat{h}(s_t) = a_t$

Limitations

- Expert's mistakes
- Mistakes of $\hat{h}$: unbounded consequences

Inverse Reinforcement Learning

Abbeel, Ng, 2004

Input

- Traces (s_t, a_t) of expert

Supervised learning

- Learn V t.q. $V(s_t, a_t) > V(s_t, a')$

Limitations

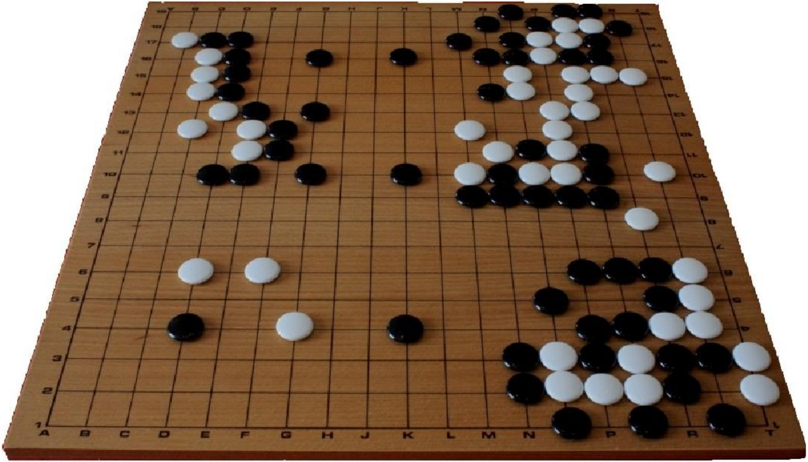
- Expert's mistakes
- Requires appropriate representation

more ?

http://videolectures.net/ecmlpkdd2012_abbeel_learning_robotics/

Overview

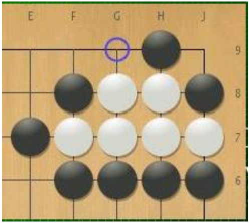
Go as AI Challenge



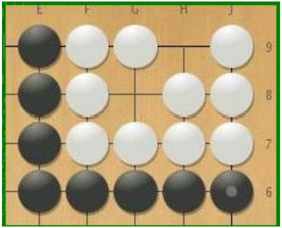
Go as AI Challenge, foll'd

Rules

- Each player puts a stone on the goban, black first
- Each stone remains on the goban, except:



group w/o degree freedom is killed



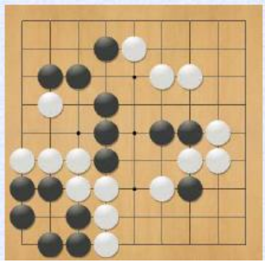
a group with two eyes can't be killed

- The goal is to control the max. territory

Go as AI Challenge, foll'd

Features

- Number of games $2.10^{170} \sim$ number of atoms in universe.
- Branching factor: 200 ($\sim$ 30 for chess)
- No good heuristic function to assess a position
- Local and global features (symmetries, freedom, ...)
- A move might make a difference some dozen plies later



Where is the difficulty ?

- You can't grow the full tree
- You can't safely cut branches
- You can't be greedy

Principles of MoGo

Gelly Wang 07, Gelly Silver 07

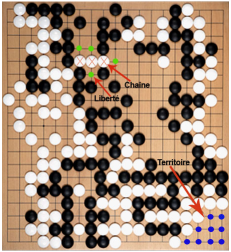
- A weak but unbiased assessment function: Monte Carlo-based
- Allowing the machine to play against itself and build its own strategy

Exploration vs Exploitation dilemma

Assessing a position   Fast and frugal...

Monte-Carlo-based   Brügman (1993)

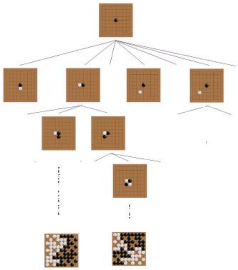
1. While possible, add a stone (white, black)
2. Compute Win(black)
3. Repeat and average



Remark: The point is to be unbiased

if there exist situations where you (wrongly) think you are in good shape
then you go there
and you are in bad shape...

Build a strategy: Monte-Carlo Tree Search



In a given situation:

Select a move — Multi-Armed Bandit

In the end:

1. Assess the final move — Monte-Carlo
2. Update reward for all moves

Select a move

Exploration vs **Exploitation**
Dilemma

Multi-Armed Bandits Lai, Robbins 85

- In a casino, one wants to maximize one's gains *while playing*
- Play the best arms so far ? **Exploitation**
- But there might exist better arms... **Exploration**

Multi-Armed Bandits, foll'd

Auer et al. 01, 02; Kocsis Szepesvári 06 **For each arm (move)**

- Reward: Bernoulli variable $\sim \mu_i, 0 \leq \mu_i \leq 1$
- Empirical estimate: $\hat{\mu}_i \pm$ Confidence (n_i) *nb trials*

Decision: Optimism in front of unknown!

$$\text{Select } i^* = \text{argmax } \hat{\mu}_i + C\sqrt{\frac{log(\sum n_j)}{n_i}}$$

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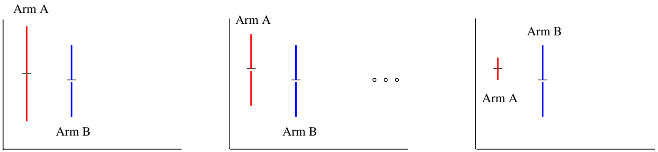


Multi-Armed Bandits, foll'd

Auer et al. 01, 02; Kocsis Szepesvári 06 **Criterion: the regret of your strategy π**

- Not what you gain,
- But what you loose compared to the oracle

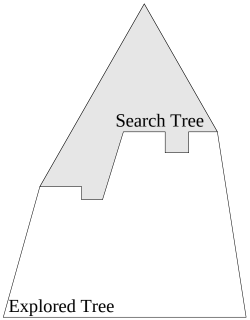
$$Regret(\pi) = \sum_{t=1}^{T} (\mu^* - \mu(\pi(t))$$

Note

- Optimal regret in log(T) Lai Robbins 85
- UCB achieves the optimal regret Auer et al. 02
- Compare to ϵ-greedy...

The UCT scheme

- Upper Confidence Tree (UCT) [1]
  - Gradually grow the search tree
  - Building Blocks
    - Select next action (bandit-based phase)
    - Add a node (leaf of the search tree)
    - Select next action bis (random phase)
    - Compute instant reward
    - Update information in visited nodes
  - Returned solution:
    - Path visited most often



L. Kocsis, and C. Szepesvári, 06

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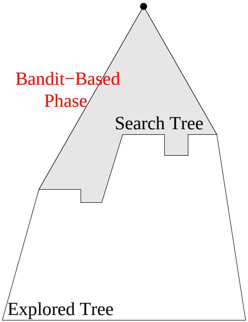

L. Kocsis, and C. Szepesvári, 06

The UCT scheme

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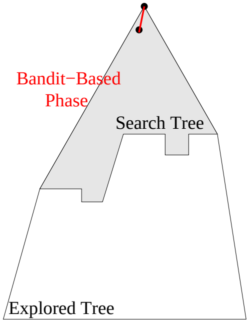

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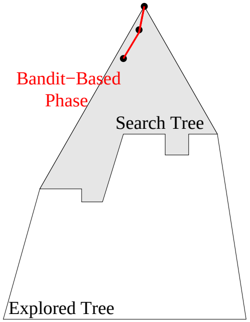

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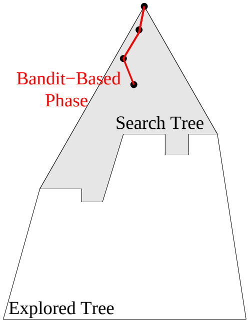

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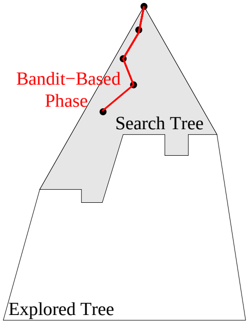

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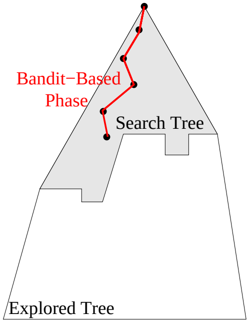

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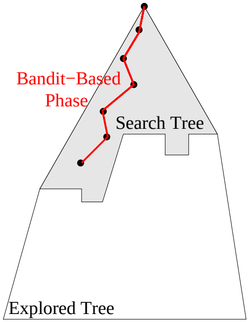

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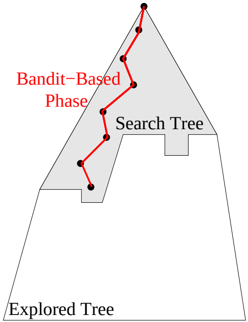

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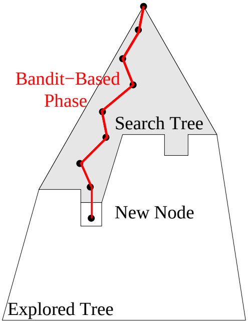

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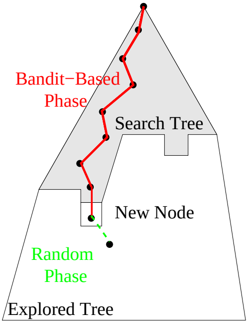

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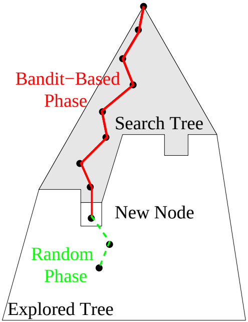

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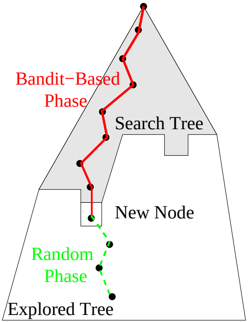

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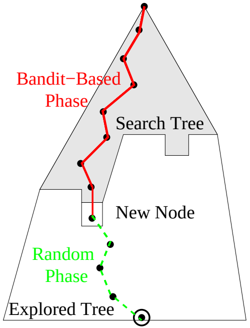

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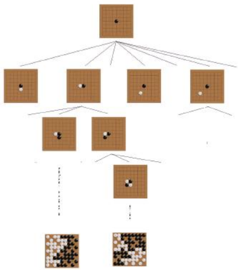
Monte-Carlo Tree Search



Comments: MCTS grows an asymmetrical tree

- Most promising branches are more explored
- thus their assessment becomes more precise
- Guarantees of optimality

Going Beyond

- Take into account standard deviation of μ
- Adjust constant C
- Needs heuristics to deal with many arms
- Share information among branches

BB1: Select Next Action

Rules for the Multi-Armed Bandit phase

Auer et al. 02

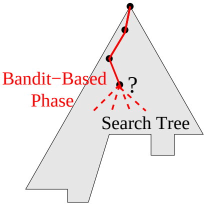
- Upper Confidence Bound
  - Select $\underset{a \in A}{argmax}\ \hat{\mu}_a + \sqrt{\frac{c_e \log(T)}{n_a}}$
- Upper Confidence Bound (UCB1-tuned)
  - Select $\underset{a \in A}{argmax}\ \hat{\mu}_a + \sqrt{\frac{c_e \log(T)}{n_a} min\left(\frac{1}{4}, \hat{\sigma}_a^2 + \sqrt{\frac{c_e \log(T)}{t_a}}\right)}$

T Total number of trials in current node

n_a Number of trials for action a

$\hat{\mu}_a$ Empirical average reward for action a

$\hat{\sigma}_a^2$ Empirical variance of reward for action a



BB1: Select Next Action, foll'd

Select $\underset{a \in A}{argmax}\ \hat{\mu}_a + \sqrt{\frac{c_e \log(T)}{n_a}}$

- Asymptotically optimal
- But visits the tree infinitely often !

Why not selecting always the best ?

- Not consistent:
  if unlucky, good arm gets 0 first time it's played, bad arm gets 1 first time it's played, always play bad arm...

Frugal and consistent

$$\text{Select } \underset{a \in A}{argmax}\ \frac{\text{Nb win} + 1}{\text{Nb loss} + 2}$$

Berthier et al. 2010

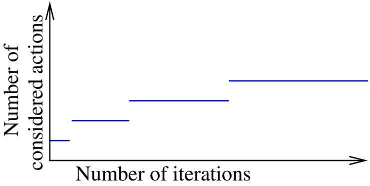
BB2: Control the branching factor

What if many arms ? degenerate into exploration

- Continuous heuristics
  Use a small exploration constant c_e
- Discrete heuristics Progressive Widening

  Coulom 06; Rolet et al. 09

  Consider only $\lfloor T^b \rfloor$ actions ($b < 1$, usually $b = 1/2$)



When $n_a = 4, 9, 16, 25, \ldots$, you are allowed to consider a new arm.

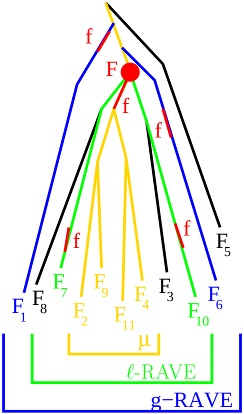
BB3: Consider a new arm, which one ?

Prefer good initialization: it takes time to recover from a bad one
Which good moves ? Sharing information among nodes

Rapid Action Value Estimation (RAVE)

Gelly Silver 07

$$\text{RAVE(n,m)} = \text{average}_{m \ ancestor \ of \ n} \ \mu(n)$$



$\hat{\mu}_a$ has a high variance $\rightarrow$ rather use

$$\alpha\hat{\mu}_a + (1-\alpha)\left(\beta RAVE_\ell(a) + (1-\beta)RAVE_g(a)\right)$$

$$\alpha = \frac{n_a}{n_a + c_1} \qquad \beta = \frac{n_{a,\ell}}{n_{a,\ell} + c_2}$$

BB4: The random phase

Note

- MoGo worked because of the smart random phase,
- not because of UCT.

Random phase, 3 variants

1. Put stones randomly
2. Put stones randomly in the neighborhoof of a previous stone
3. Put stones matching patterns   expertise

Many trials !

Failures

- Grafting expert knowledge
- Replacing the random phase by a smarter one
- Using value of the game instead of 0/1
- Compiling RAVE with a neural net

Successes

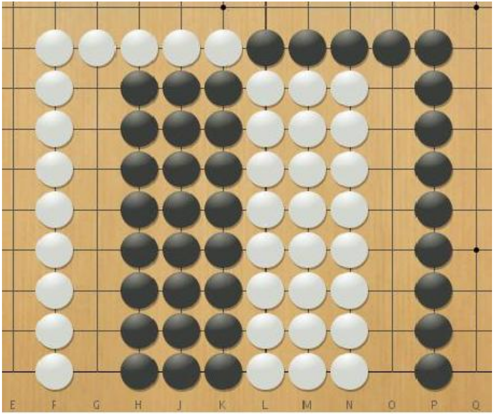
- Parallelizing
- Learning an opening book

Comparative results

2011	First win against a pro (6D), H2, 13×13	MoGoTW
2011	First win against a pro (9P), H2.5, 13×13	MoGoTW
2011	First win against a pro in Blind Go, 9×9	MoGoTW
2010	Gold medal in TAAI, all categories 19×19, 13×13, 9×9	MoGoTW
2007	Win against a pro (5P), 9× 9 (blitz)	MoGo
2008	in against a pro (5P), 9× 9 (white)	MoGo
2009	Win against a pro (5P), 9× 9 (black)	MoGo
2009	Win against a pro (5P), 9× 9 (black)	MoGoTW
200	Win against a pro (8P), 19× 19 H9	MoGo
2008	Win against a pro (9P), 19× 19 H7	MoGo
2009	Win against a pro (1P), 19× 19 H6	MoGo

Comparative results

Failure: Semeai



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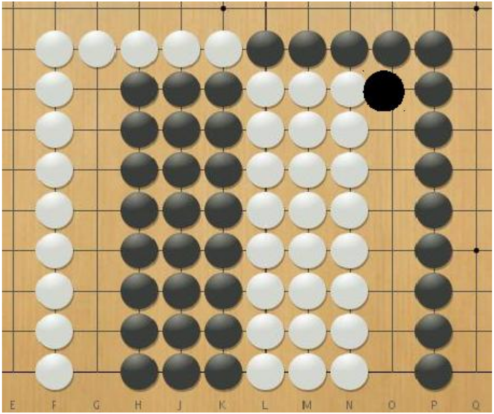

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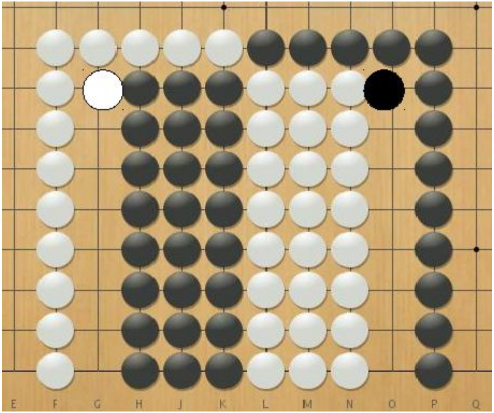

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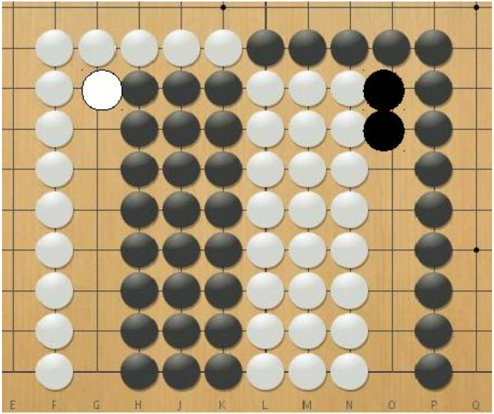

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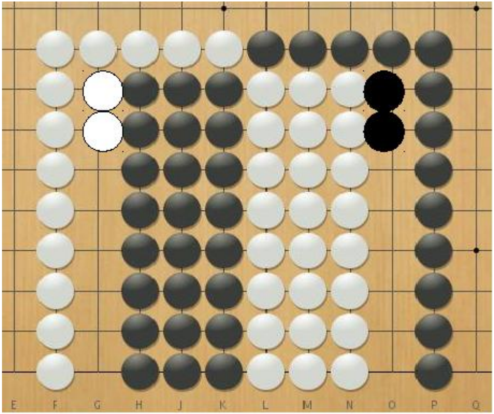

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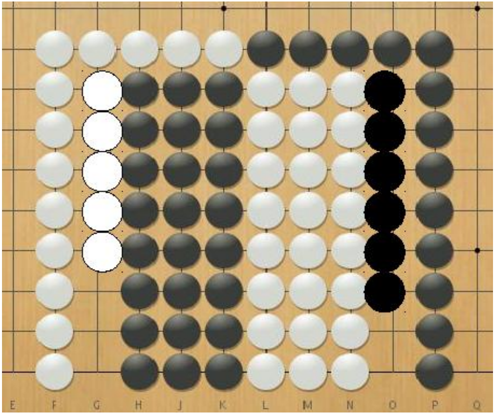

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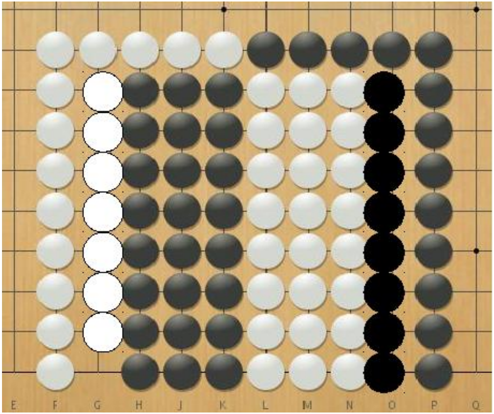

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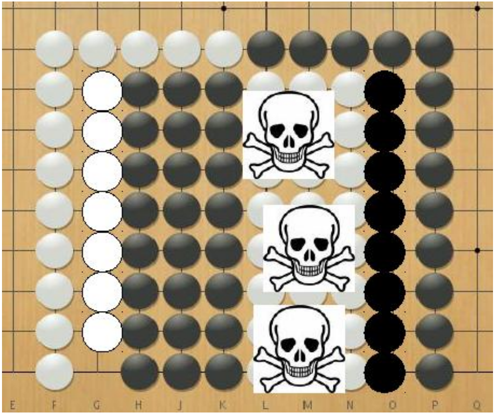

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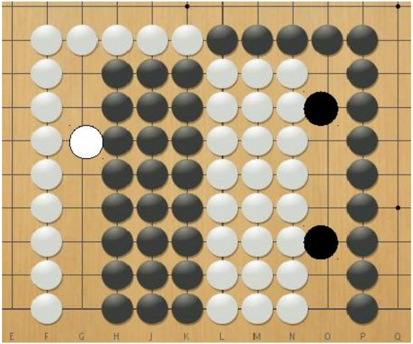
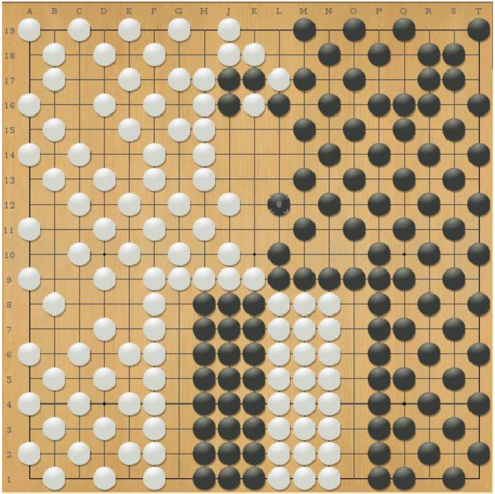


Why does it fail

- First simulation gives 50%
- Following simulations give 100% or 0%
- But MCTS tries other moves: doesn't see all moves on the black side are equivalent.

%

Implications



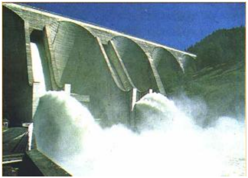
Energy policy

Claim

Many problems can be phrased as optimization in front of the uncertainty.

Adversarial setting	2 two-player game
uniform setting	a single player game

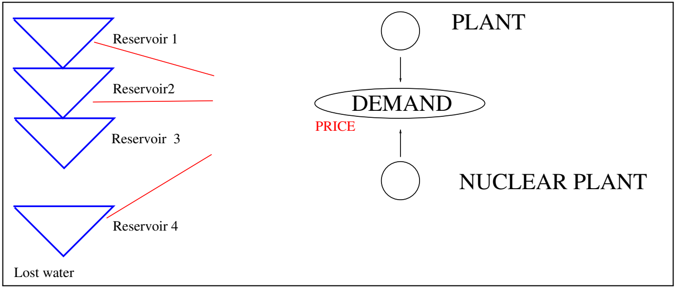
Management of energy stocks under uncertainty

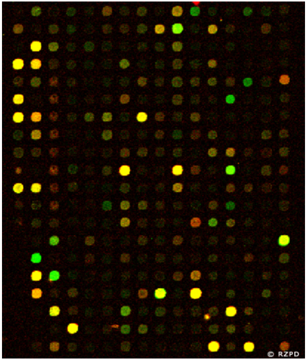

States and Decisions

States

- Amount of stock (60 nuclear, 20 hydro.)
- Varying: price, weather — alea or archive
- Decision: release water from one reservoir to another
- Assessment: meet the demand, otherwise buy energy



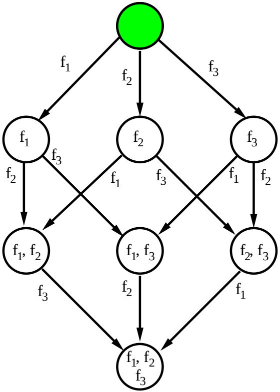
When Machine Learning is Feature Selection

Bio-informatics 

- 30 000 genes
- few examples (expensive)
- Goal: find the genes related to (disease, exposure to radiations, etc).

Feature Selection

Gaudel Sebag, 10



Combinatorial optimization:

Set of features $\mathcal{F}$

Set of states $\mathcal{S} = 2^{\mathcal{F}}$

Initial state $\emptyset$

Set of actions $A = \{\text{add } f, \ f \in \mathcal{F}\}$

Final state any state

Reward function $r : \mathcal{S} \mapsto [0, 1]$

Goal: minimize generalization error

$$\text{Find } \underset{F \subseteq \mathcal{F}}{argmin} \ \mathbf{Err}\left(\mathcal{A}\left(F, D\right)\right)$$

Feature Selection with MCTS: Fuse

Select next move: UCB + Progressive widening

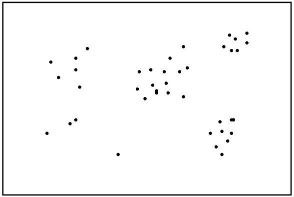
Select new arm/feature: use RAVE

Random phase

- No fixed time horizon
- Introduce specific stopping actions
- In random phase, stop with probability $1 - q^d$ ($q < 1$, user specified)

FUSE: reward(F)
Generalization error estimate

- Requisite
  - fast (to be computed 10^4 times)
  - unbiased
- Proposed reward
  - k-NN like
  - + AUC criterion *
- Complexity: $\tilde{O}(mnd)$



d Number of selected features
n Size of the training set
m Size of sub-sample ($m \ll n$)

(*) Mann Whitney Wilcoxon test:

$$V(F) = \frac{|\{((x,y),(x',y')) \in \mathcal{V}^2,\ \mathcal{N}_{F,k}(x) < \mathcal{N}_{F,k}(x'),\ y < y'\}|}{|\{((x,y),(x',y')) \in \mathcal{V}^2,\ y < y'\}|}$$

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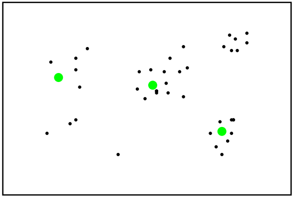

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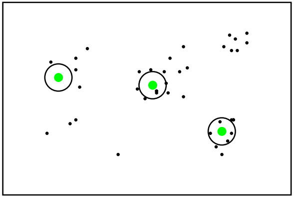

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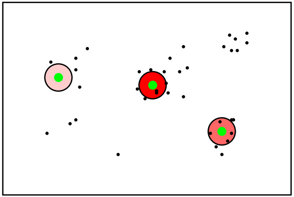

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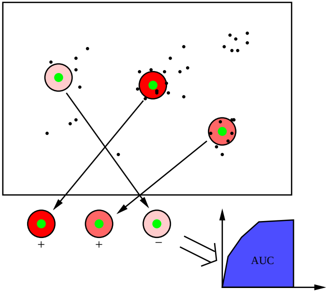

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Experimental setting

- Questions
  - FUSE vs FUSE^R
  - Continuous vs discrete exploration heuristics
  - FS performance w.r.t. complexity of the target concept
  - Convergence speed
- Experiments on

Data set	Samples	Features	Properties
Madelon [1]	2,600	500	XOR-like
Arcene [1]	200	10,000	Redundant features
Colon	62	2,000	"Easy"

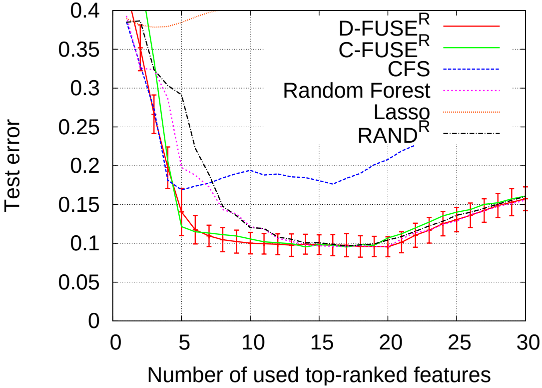
[1] NIPS'03

Experimental setting

- Baselines
  - CFS (Constraint-based Feature Selection) [1]
  - Random Forest [2]
  - Lasso [3]
  - RAND^R: RAVE obtained by selecting 20 random features at each iteration
- Results averaged on 50 splits (10 $\times$ 5 fold cross-validation)
- End learner
  - Hyper-parameters optimized by 5 fold cross-validation

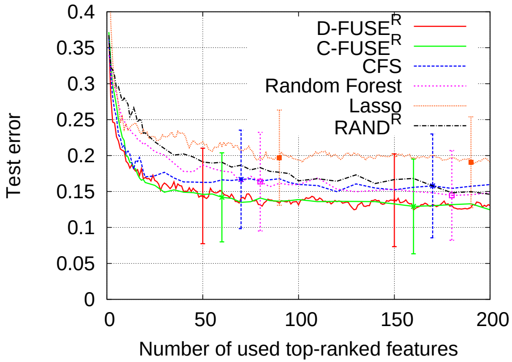
[1] M. A. Hall ICML'00
[2] J. Rogers, and S. R. Gunn SLSFS'05
[3] R. Tibshirani Journal of the Royal Statistical Society 94

Results on Madelon after 200,000 iterations



- Remark: FUSE^R = best of both worlds
  - Removes redundancy (like CFS)
  - Keeps conditionally relevant features (like Random Forest)

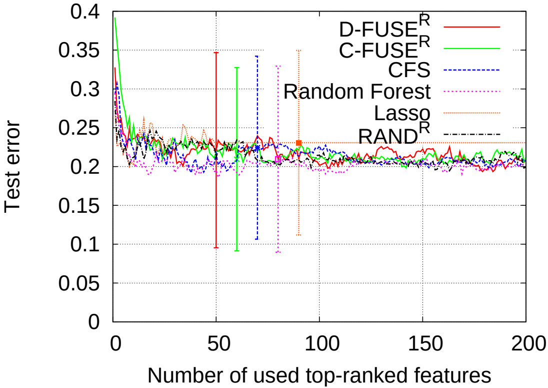
Results on Arcene after 200,000 iterations



- Remark: FUSE^R $=$ best of both worlds
  - Removes redundancy (like CFS)
  - Keeps conditionally relevant features (like Random Forest)

[0] T-test "CFS vs. FUSE^R" with 100 features: p-value=0.036

Results on Colon after 200,000 iterations



- Remark
  - All equivalent

NIPS 2003 Feature Selection challenge

- Test error on a disjoint test set

DATABASE	ALGORITHM	CHALLENGE ERROR	SUBMITTED FEATURES	IRRELEVANT FEATURES
MADELON	FSPP2 [1]	6.22% (1^{st})	12	0
	D-FUSE^R	6.50% (24^{th})	18	0
	BAYES-NN-RED [2]	7.20% (1^{st})	100	0
ARCENE	D-FUSE^R(ON ALL)	8.42% (3^{rd})	500	34
	D-FUSE^R	9.42% 500 (8^{th})	500	0

[1] K. Q. Shen, C. J. Ong, X. P. Li, E. P. V. Wilder-Smith Mach. Learn. 2008
[2] R. M. Neal, and J. Zhang Feature extraction, foundations and applications, Springer 2006

Conclusion

Contributions

- Formalization of Feature Selection as a Markov Decision Process
- Efficient approximation of the optimal policy (based on UCT)
  $\Rightarrow$ Any-time algorithm
- Experimental results
  - State of the art
  - High computational cost (45 minutes on Madelon)

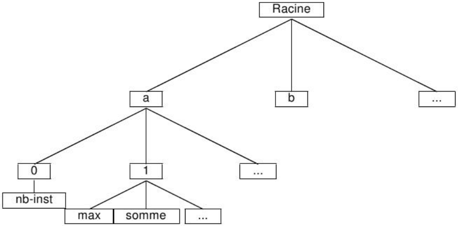
Perspectives

- Use other end learners
- Revisit the reward see (Hand 2010) about AUC
- Extend to Feature construction

Extension to Feature Construction

Context: Data Mining from Relational Databases

- A star schema (customer, invoices, services...)
- Usually: propositionalize the database
- Alternative: Feature Construction
  Explore the language of queries



Lesson1: When is MCTS relevant

Applicable when

- High dimension
- Complex model
- Delayed reward

Challenges

- When the game is highly non-observable
- When no baseline for the random phase

Lesson 2: Using by-products of MCTS

- FUSE with N iterations:
  each iteration i) follows a path; ii) evaluates a final node
- Result:

Search tree (most visited path)	$\longleftrightarrow$	RAVE score
$\Downarrow$		$\Downarrow$
Wrapper approach		**Filter approach**
FUSE		**FUSE**^{R}

- On the feature subset, use end learner $\mathcal{A}$
  - Any Machine Learning algorithm
  - Support Vector Machine with Gaussian kernel in experiments

Perspectives

Extensions

- Continuous RAVE (continuous action / state spaces)
- Partially observable settings (poker)

Applications

- Feature construction
- Robotics (action selection)