Functional Synchronous Programming of Reactive Systems

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Embedded Software

Evolution since the 70's

- Moore law: the computing power double every 18 months
- 2006: 500 000 000 transistors (Pentium), 4Ghz
- Especially: small dedicated processors everywhere!
  - consumer electronics: TV, video, mobile phones
  - automotive: electronic systems, ABS, braking systems, electronic key
- dedicated circuits (ASIC), dedicated or general purpose computers executing software
- now: dynamic reconfiguration, cold (ADSL modem) or hot (industrial systems, mobile phones)

A progressive transition

- first in **industrial fields**
  - replacing mechanical, electro-mechanical, electronic systems
  - from relay systems (train) to software systems
  - flight commands (from direct mechanics, electro-mechanic support (Caravelle), analog systems (Concorde) to numerical and software (Airbus 320, 1988))
  - regulation systems, control/command of industrial processes
- **almost everywhere now**
  - heater controller, washing machine, TV boxes, etc.
- and it is possible to **simulate** the whole before building it

Embedded real-time systems: characteristics

Hard Real-time Constraints

- the system is submitted to hard real-time constraints
- **imposed by the environment** (physics does not wait!)

$\hookrightarrow$ (statically) bounded response time and memory

Heterogeneous environment

- **continuous** physical environment: temperature sensors, activators
- and/or **discrete**: button, threshold, other computers
- a huge number of input/outputs

$\hookrightarrow$ the formalism must be able to express this heterogeneity

Concurrent and Deterministic Systems

- **closed loop** systems: a heater controller and the heater itself run in parallel
- concurrency is the natural way for **composing systems**: *control at the same time rolling and pitching*
- the whole system must stay **deterministic**
- also **simpler**: reproducibility, easier debugging, simulation

↪ the formalism must be able to compose sub-systems in parallel

↪ the model must conciliate concurrency and determinism

Safety is important

- **critical** systems: fly-by-wire, braking systems, airbags, medical systems
- some systems do not have a stable (safe) position (plane?)

↪ properties must be guaranteed statically: "dynamic" = "too late"

↪ languages with a well founded semantics, formal validation

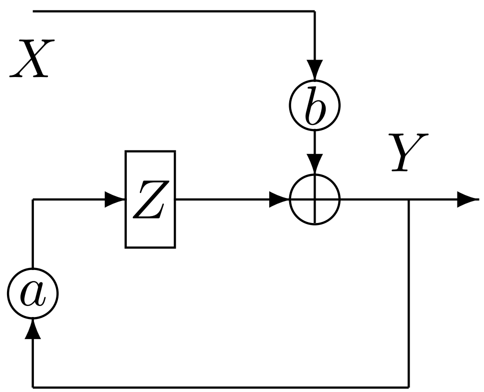
Design Domain Specific Languages

- the general-purpose software model is in-adapted: Turing complete, too expressive, too hard to verify
- complexity is not where it is needed: pointer arithmetics, dynamic allocation, etc.
- do we need all that?
- concurrency and determinism are absent but there are fundamental!
- far from the mathematical model of the engineer
- design specific languages with a limited expressive power, a formal semantics, well adapted to the culture of the field
- this culture is rich:
  - continuous control (control theory, sampling, etc.)
  - discrete control (automata theory, etc.)

Continuous control (control theory, signal processing...)

- a signal/event is represented by a discrete sequence (a *stream*)
  $\hookrightarrow$ *stream equations, generating functions, z-transform*
  $\hookrightarrow$ *graphical formalism (block-diagrams)*
- manual transcription of these equations into imperative programs
- hard and error-prone

$$Y_0 = bX_0 \;,\; \forall n \; Y_{n+1} = aY_n + bX_{n+1}$$

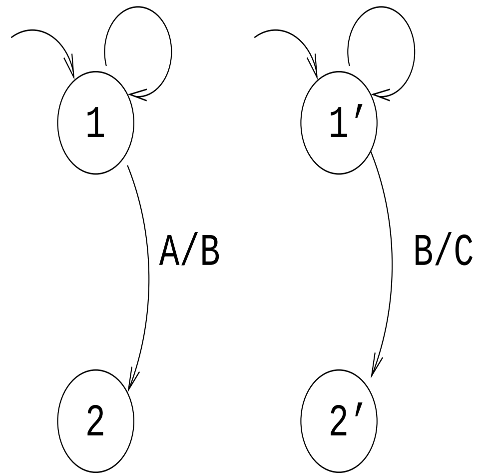


The idea of Lustre (and Signal) (1984):

- program directly with these stream equations
- provide a compiler and tool analysis

Discrete Control

- transition systems (automata), Mealy/Moore machines
- synchronous composition of automata + hierarchy, etc.
- process calculi (Milner's SCCS)

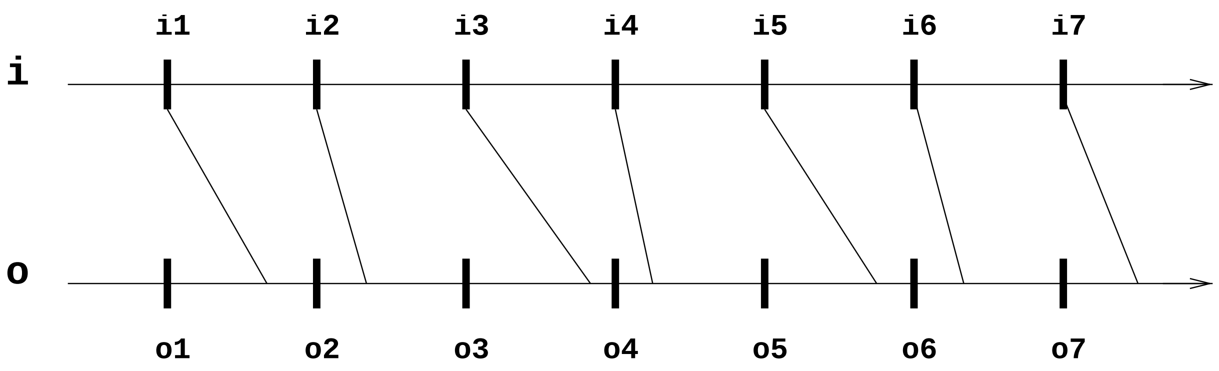


Esterel (82):

- propose a high-level language for concurrent systems
- based on the synchronous composition
- preserving determinism (causality problems are solved)

The Synchronous Model of Time

- these languages are based on the **zero delay** model
  - time is **logical** as the sequence of atomic reaction of the system to input events
  - the system is the **parallel composition** of sub-systems which (virtually) execute in parallel
  - check afterwoods the **correspondence** between logical time and physical time: is the machine fast enough?



maximum response time $max_{n \in IN}(t_n - t_{n-1}) \leq bound$

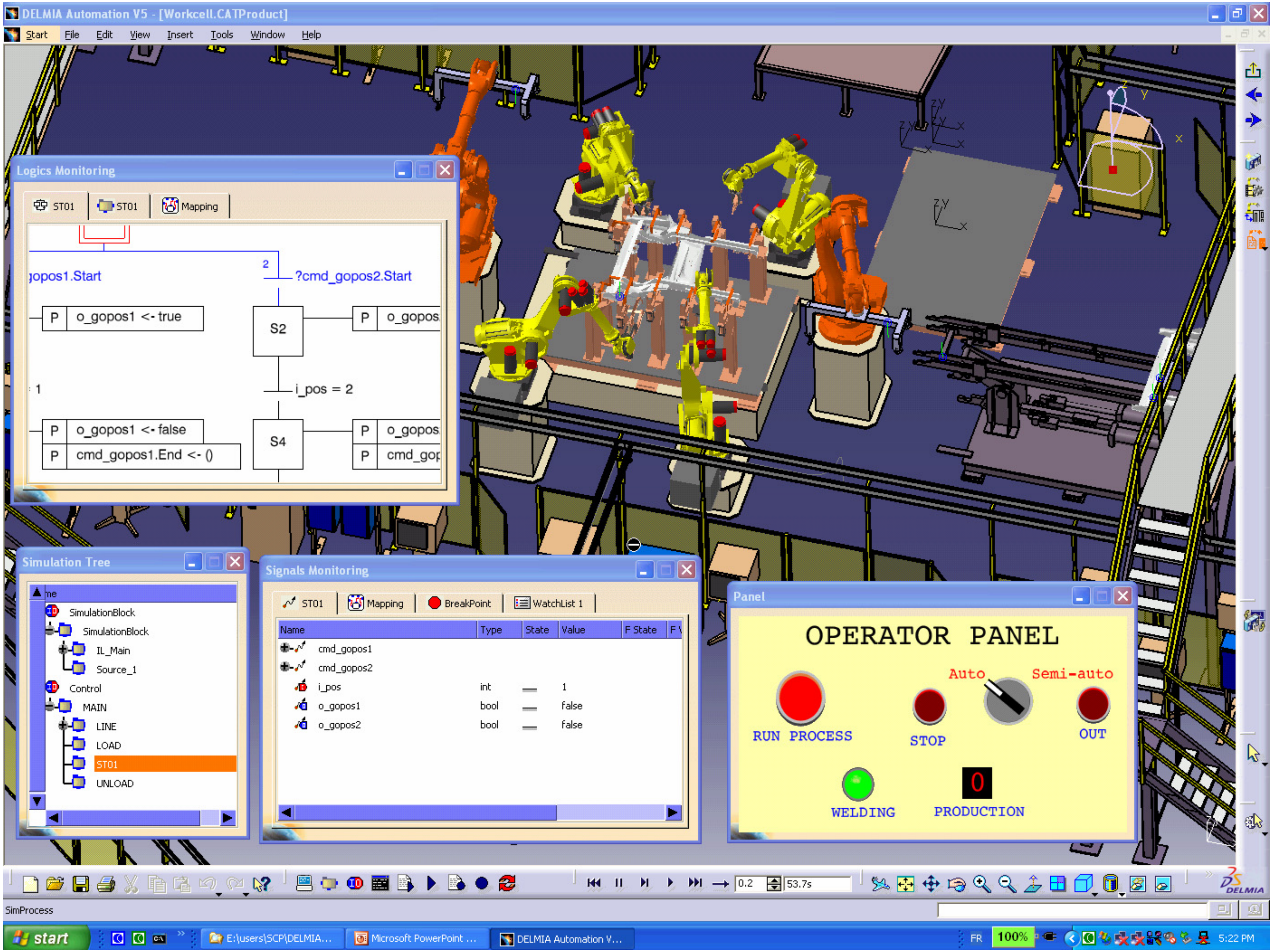
Synchronous Languages

- based on a common model but with different programming styles
- imperative (automata): **Esterel**, **SyncCharts**, **Argos**
- declarative (dataflow): **Lustre**, **Signal**
- industrial compiler (and environment) SCADE/Lustre, Esterel-studio (Esterel-Technologies), Signal/Sildex (TNI)
- formal semantics, hardware and software compilation of the same description, test tools and automatic verification
- automatic distribution (distributed architectures), (real-time) multi-tasking, etc.
- industrial succes: avionics (Airbus, Dassault, Eurocopter), ground transportation (Matra, Audi), circuits (Xilink, TI, Intel)

But also...

- simulation tools: Simulink/StateFlow (The MathWorks), Catia (Dassault-Systèmes), etc.
- very rich plateform to simulate the whole system **and** its environment
- based on numerical analysis techniques, simulation techniques
- (partial) code generation, verification tools, etc.
- these tools are not that far from synchronous tools
- block-diagram description *à la SCADE* with Simulink; state-transition systems *à la SyncCharts* with StateFlow...
- but they have not been designed with a programming discipline in mind (where what is executed is exactly what is modeled and simulated)
- informal semantics (code certification, good code quality?), formal proof/verification tools?
- what is is simulated and what is executed must be the same!

Software Factory with Catia + LCM (Dassault-Systèmes)



Needs for Synchronous Tools and Models

- master the complexity and large scale systems
  - what to do with all these transistors?
  - critical systems become big: 500 000 lines of code for the fly-by-wire command of the A380
  - some companies only specify the system and assemble the code made by others
- modularity (libraries), abstraction mechanism
- "langage" tools (vs verification) which give guaranty at compile time: automatic type and clock inference (**mandatory** in a graphical environment), deadlock freedom, etc.
- how to combine dataflow (e.g., Lustre) and control-flow (e.g., Esterel) in a uniform way?
- links with tools for formal certification (code certification is mandatory in civil avionics, DO 178B norm)
- code certification, link with proof assistant

The origin of Lucid Synchrone

In 95, with Paul Caspi (VERIMAG)

What are the relationships between:

- Kahn process networks
- synchronous data-flow programming (e.g., Lustre)
- tools and models of control theory/signal processing
- lazy functional programming (e.g., Haskell)
- types and clocks
- state machines and stream functions

What can we learn from bringing together synchronous programming and functional programming?

Lucid Synchrone

Build a “laboratory” language

- study the extensions of Lustre (synchronous and functional)
- experiment things and write programs!
- Version 1 (1995), Version 2 (2001), V3 (2006)

Semantics

- Synchronous Kahn networks [ICFP'96]
- Clocks as dependent types [ICFP'96]
- synchronous stream functions and transition systems (co-induction *vs* co-itération) [CMCS'98]
- ML-like clock calculus [Emsoft'03]
- causality analysis [ESOP'01]
- initialization analysis [SLAP'03, STTT'04]
- higher-order and typing [Emsoft'04]
- data-flow and state machines [Emsoft'05]
- N-Synchronous Kahn Networks [Emsoft'05, POPL'06]

Some examples (V3)

- `int` denote the type of streams of integers,
- `1` denotes an (infinite) constant stream of 1,
- usual primitives apply point-wise

`c`	t	f	t	$\ldots$
`x`	x_0	x_1	x_2	$\ldots$
`y`	y_0	y_1	y_2	$\ldots$
`if c then x else y`	x_0	y_1	x_2	$\ldots$

Combinatorial functions

Example: 1-bit adder

```
let xor x y = (x & not (y)) or (not x & y)

let full_add(a, b, c) = (s, co)
  where
      s = (a xor b) xor c
  and co = (a & b) or (b & c) or (a & c)
```

The compiler automatically computes the type and clock signature.

```
val full_add :  bool * bool * bool -> bool * bool
val full_add :: 'a * 'a * 'a -> 'a * 'a
```

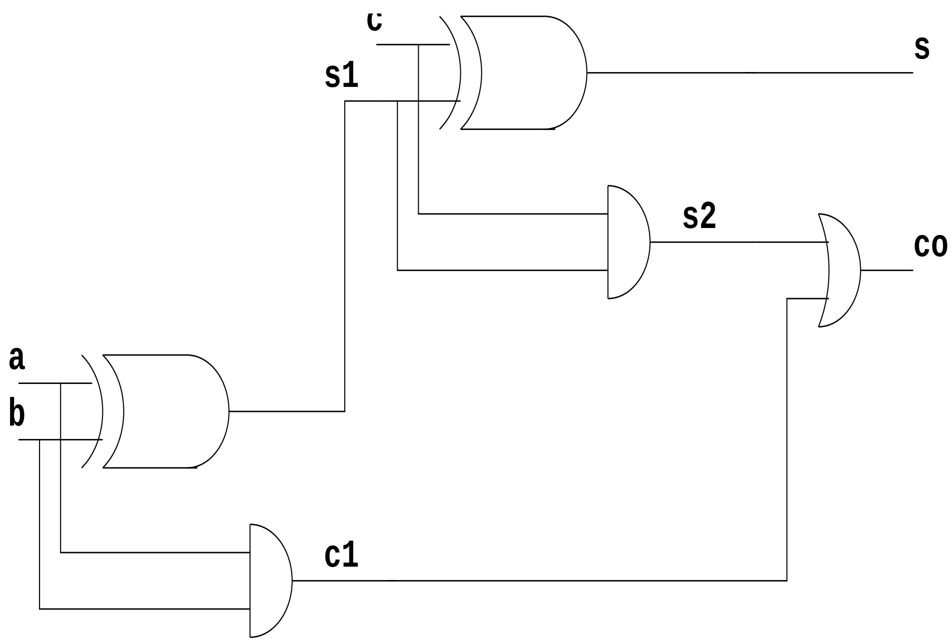
Full Adder (hierarchical)

Compose two “half adder”

```
let half_add(a,b) = (s, co)
   where
       s = a xor b
   and co = a & b
```

Instanciate twice

```
let full_add(a,b,c) = (s, co)
  where
  rec (s1, c1) = half_add(a,b)
  and (s, c2) = half_add(c, s1)
  and co = c1 or c2
```



Sequential Functions

Operators `fby`, `->`, `pre`

- `fby`: unitary (initialized) delay
- `->`: initialization
- `pre`: un-initialized delay (register in circuits)

`x`	x_0	x_1	x_2	$\ldots$
`y`	y_0	y_1	y_2	$\ldots$
`x fby y`	x_0	y_0	y_1	$\ldots$
`pre x`	nil	x_0	x_1	$\ldots$
`x -> y`	x_0	y_1	y_2	$\ldots$

Sequential Functions

- Stream functions may depend on the past (statefull systems)
- Example: edge front detector

```
let node edge x = x -> not (pre x) & x


val sum :  int => int
val sum :: 'a -> 'a
```

`x`	t	f	t	t	t	f	...
`edge x`	t	f	t	f	f	f	...

In V3, we distinguish combinatorial function (`->`) from sequential functions (`=>`)

Polymorphism (code reuse)

```
let node delay x = x -> pre x

val delay :  'a => 'a
val delay :: 'a -> 'a

let node edge x = false -> x <> pre x

val edge :  'a => 'a
val edge :: 'a -> 'a
```

In Lustre, polymorphism is limited to a set of predefined operators (e.g., `if/then/else`, `when`) and does not pass abstraction.

Library and Curryfication

```
(* module Numerical *)
let node integr dt x0 dx = x where
  rec x = x0 -> pre x +. dx *. dt

val integr :  float -> float -> float => float
val integr :: 'a -> 'a -> 'a -> 'a



(* module Main *)
let static dt = 0.001

let integr = integr dt

val integr :  float -> float => float
val integr :: 'a -> 'a -> 'a
```

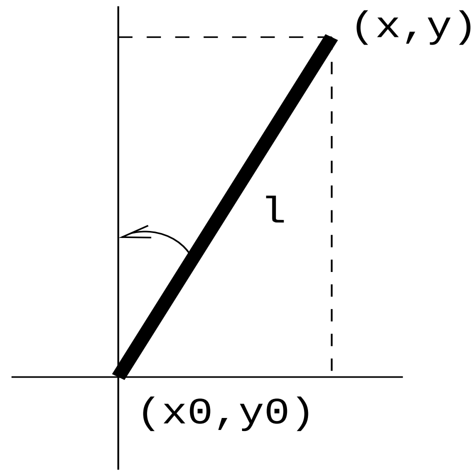
Example: the inverted pendulum

Specification: control an inverted pendulum

$$l * \frac{d^2\theta}{dt^2} = sin(\theta) * (\frac{d^2 y_0}{dt^2} + g) - (cos(\theta) * \frac{d^2 x_0}{dt^2})$$

$$x = x_0 + l.sin(\theta)$$

$$y = y_0 + l.cos(\theta)$$



Main module:

Constants:

```
let static dt = 0.001 (* sampling step *)
and static l = 10.0   (* length   *)
and static g = 9.81   (* acceleration *)

(* partial application with fixed step *)
let integr = Numerical.integr dt
let deriv = Numerical.deriv dt
```

The equation of the pendulum

```
let node pendulum d2x0 d2y0 = theta where
  rec theta = integr (integr ((sin thetap)*.(d2y0 +. g)
                            -.(cos thetap)*.d2x0)/.l)
  and thetap = 0.0 -> pre theta
```

Reject programs

Reject program which cannot be executed sequentially

```
let node pendul d2x0 d2y0 = theta
  where rec theta =
        integr (integr ((sin theta)*.(d2y0 +. g)
                             ^^^^^^
                          -.(cos theta)*.d2x0)/.l)
thetap depends instantaneously on itself
```

- a “syntactical” criteria: a recursion must cross a delay
- a type system, with Pascal Cuoq [ESOP'01]
- thus, with type signatures (interfaces)
- modular and higher-order

Reject programs

Reject program for which the result depend on the initial value of some delays

```
let node pendul d2x0 d2y0 = theta
  where rec theta =
        integr (integr ((sin (pre theta)*.(d2y0 +. g)
        ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
                          -.(cos (pre theta))*.d2x0)/.l)
                          ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^

this expression may not be initialized
```

- 1-bit abstraction
- a type system (with sub-typing rules), with JL-Colaço from Esterel-Technologies [SLAP'02, STTT'04]
- works well for SCADE
- tested on real-size examples (75000 lines) at Esterel-Tech.

Clocks: mix several time-scale

- mix slow and fast processes in a safe way?
- multi-sampled systems (software), multi-clock (hardware)
- introduced in Lustre and Signal at the very beginning
- also present in Simulink (periodic systems)

In **Lucid Synchrone**, a clock is a type and is automatically inferred

Two operators

`when` (under-sampling) and `merge` (over-sampling)

`c`	t	t	f	f	t	f	...
`x`	x_0	x_1	x_2	x_3	x_4	x_5	...
`x when c`	x_0	x_1			x_4		...
`x whenot c`			x_2	x_3		x_5	...
`y`	y_0	y_1			y_2		...
`merge c y (x whenot c)`	y_0	y_1	x_2	x_3	y_2	x_5	...

Example

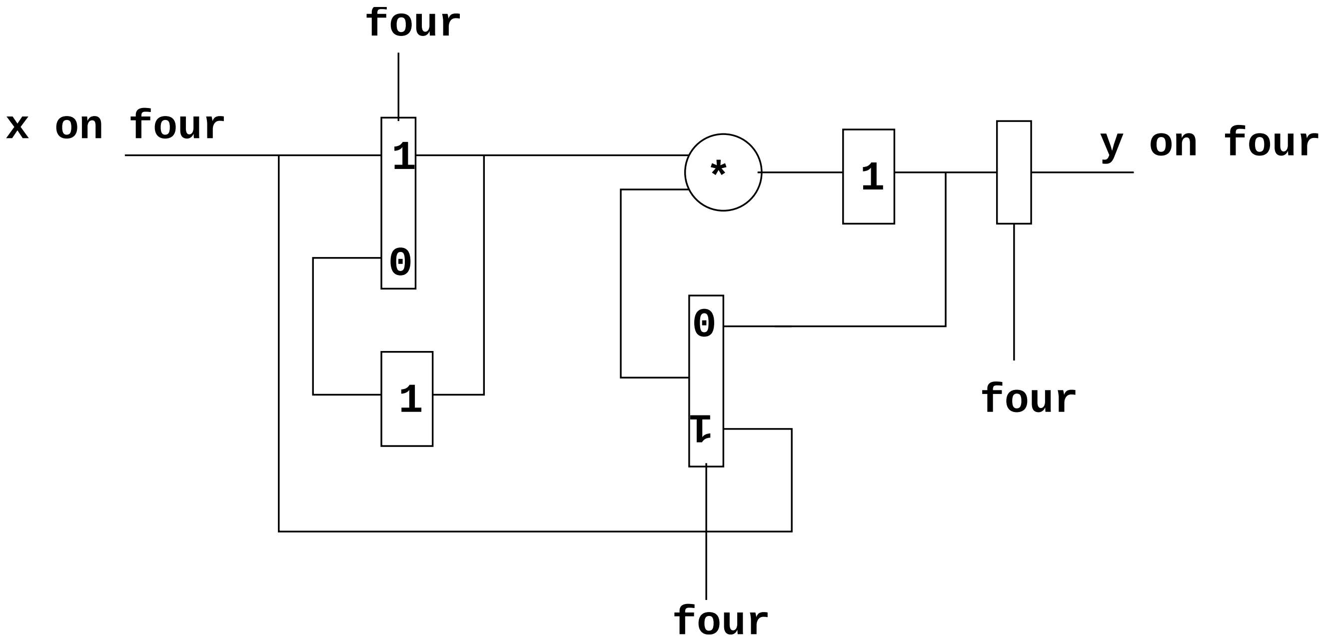
```
let node sum x = s where rec s = x -> pre s + x
let node sampled_sum x c = sum (x when c)

val sampled_sum :  int -> bool => int
val sampled_sum :: 'a -> (_c0:'a) -> 'a on _c0

let clock ten = count 10 true
let node sum_ten x = sampled_sum x ten

val ten :  bool
val ten :: 'a
val sum_ten :  int => int
val sum_ten :: 'a -> 'a on ten
```

Over-sampling

- Define systems whose internal rate is faster that the rate of their inputs?
- express temporal constraints, scheduling, resources

Example: Computation of `x^5`

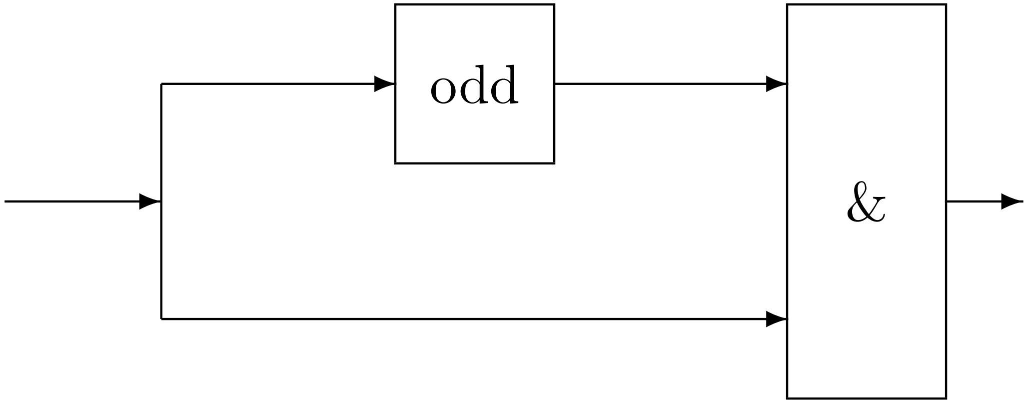
```
let node power x = x * x * x * x * x


let clock four = count 4 true
let node spower x = y where
  rec i = merge four x ((1 fby i) whenot four)
  and o = 1 fby (i * merge four x (o whenot four))
  and y = o when four

val power  :: 'a -> 'a
val spower :: 'a on four -> 'a on four
```



four	t	f	f	f	t	f	f	f	t	f	f	...
x	x_0				x_1				x_2			...
i	x_0	x_0	x_0	x_0	x_1	x_1	x_1	x_1	x_2	x_2	x_2	...
o	1	x_0^2	x_0^3	x_0^4	x_0^5	x_1^2	x_1^3	x_1^4	x_1^5	x_2^2	x_2^3	...
spower x	1				x_0^5				x_1^5			...
power x	x_0^5				x_1^5				x_2^5			...

Property: `1 fby (power x)` and `spower x` are observationally equivalent

Clock Constraints and Synchrony



The computation of $(x_n \,\&\, x_{2n})_{n \in IN}$ is not real-time

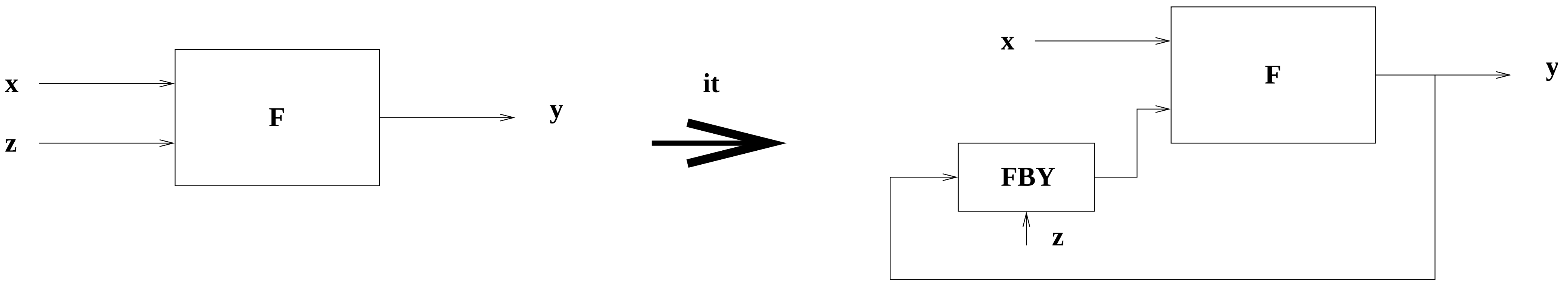
```
let odd x = x when half
let non_synchronous x =  x & (odd  x)
                             ^^^^^^^^^

This expression has clock 'a on half, but is used with clock 'a.
```

Execution with unbounded FIFOs!!!

- clocks = an information about the behavior of streams
- clocks = types
- the `merge` and type based clock calculus is reused in the ReLuC compiler of SCADE

Higher-order

Iteration:




```
let node it f z x = y
    where rec y = f x (init fby y)
```

```
val it :  ('b -> 'a -> 'a) -> 'a -> 'b => 'a
val it ::  ('b -> 'a -> 'a) -> 'a -> 'b -> 'a
```

Example:

```
let node sum x = it (+) 0 x
let node mult x = it (*) 1 x
```

Mixed systems (data-flow + automata)

Data-dominated systems: sampled systems, block-diagram formalisms

↪ Simulation tools: Simulink, etc.

↪ Programming languages: Scade/Lustre, Signal, etc.

Control dominated systems: transition systems, event systems, automata formalisms

↪ StateFlow, StateCharts

↪ SyncCharts, Argos, Esterel, etc.

What about mixed designs?

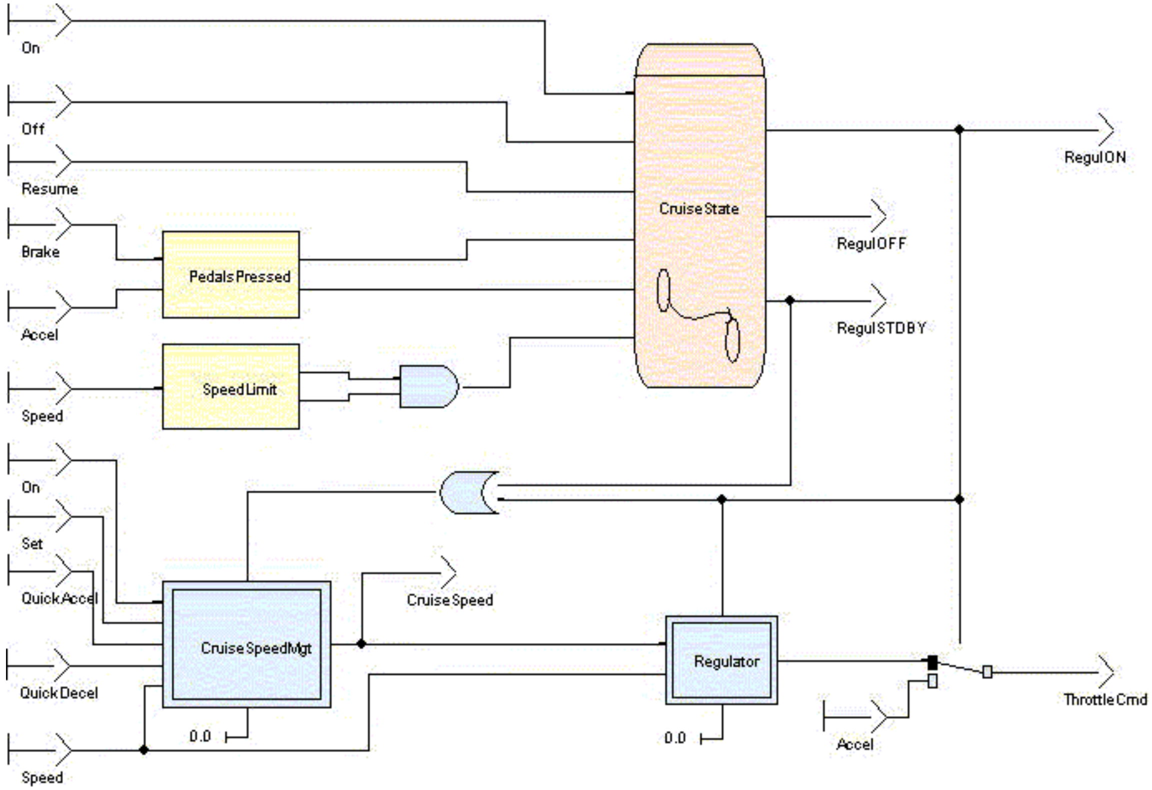
- real systems are a mix of both styles: systems have **running modes**
- each mode is defined by a control law, naturally written with data-flow equations
- a transition system for switching between these modes

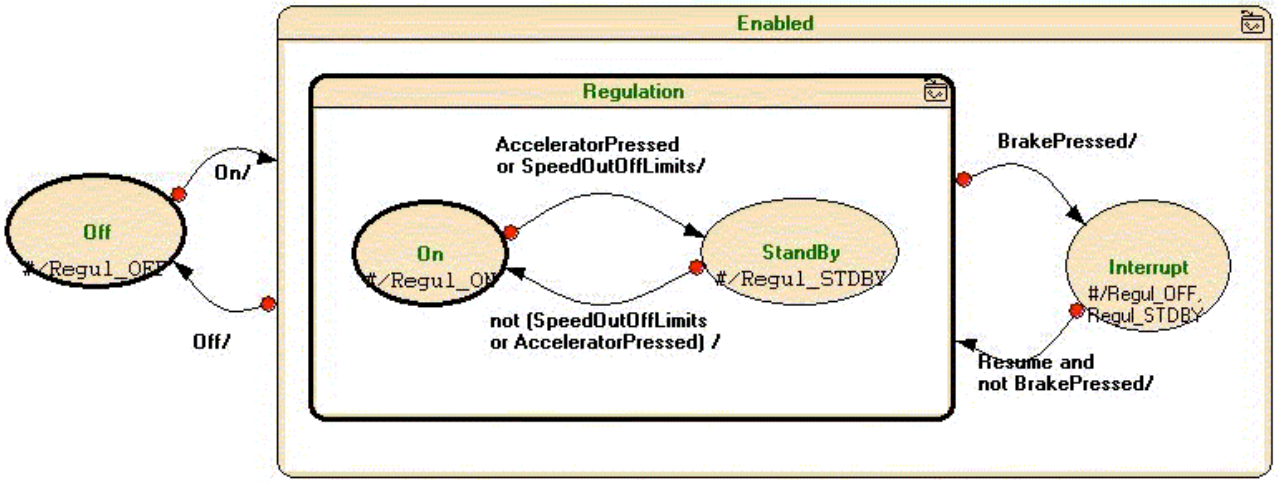
Extend SCADE/Lustre with state machines

Existing solutions

- two (or more) specific languages: one for the data part, one for the control part
- "linking" mechanisms: a sequential system is always more or less of the form
  - a transition function $f : S \times I \rightarrow O \times S$
  - an initial memory $M_0 : S$
- agree on a common representation + glue
- exist in most academic or industrial tools
- PtolemyII, Simulink + StateFlow, Lustre + Esterel Studio SSM, etc.

An example: the cruise control (SCADE V4.2)





Observations

- automata only appear at the leaves of the data-flow model: we need a finer integration
- force the programmer to make decisions at the very beginning of the design (what is the good methodology?)
- the control structure is not explicit and is hidden in boolean values: nothing say that modes are exclusive
- code certification?
- efficiency/simplicity of the code?
- how to exploit this information in static analysis and verification tools?

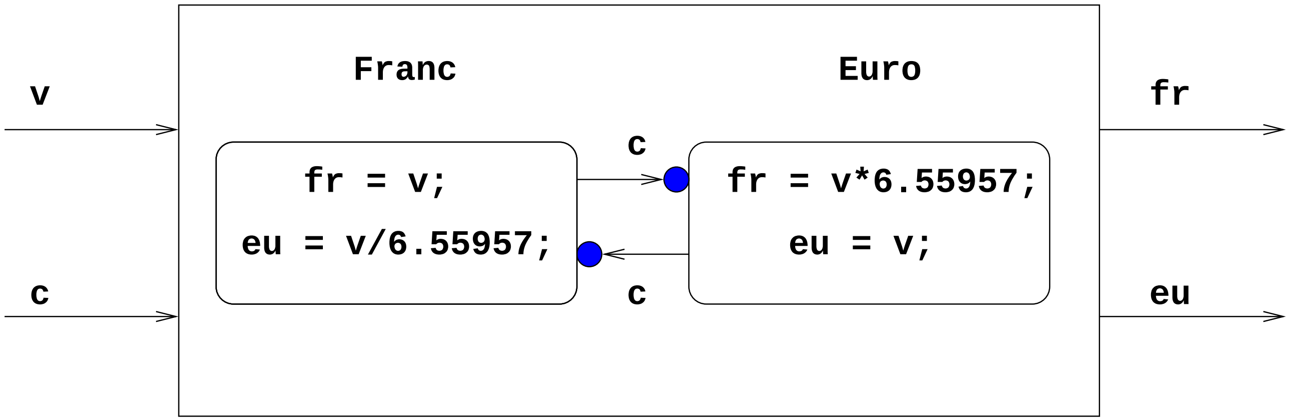
The Approach

- extend a synchronous data-flow language (Lustre) with automata constructs
- base it on a unified theory of synchronous systems
- produce efficient code (which compete with ad-hoc techniques)
- efficient compilation techniques, conservative (accept all SCADE/Lustre)

Two implementations

- ReLuC compiler of SCADE at Esterel-Tech.
- Lucid Synchrone V3

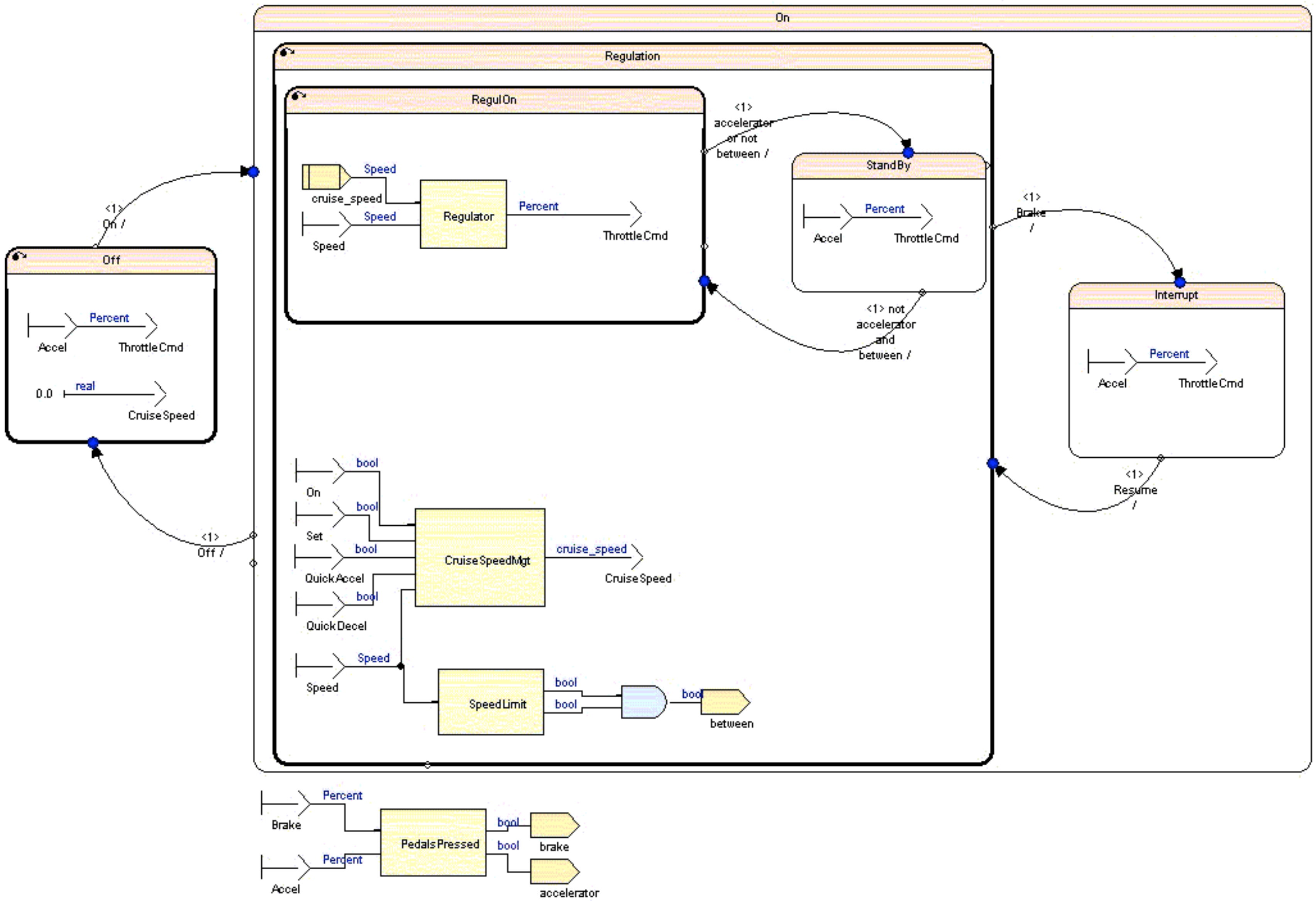
A simple example: the Franc/Euro converter



In **Lucid Synchrone** syntax:

```
let node converter v c = (euro, fr) where
  automaton
    Franc -> do fr = v and eur = v / 6.55957
             until c then Euro
  | Euro -> do fr = v * 6.55957 and eu = v
            until c then Franc
  end
```

The Cruise Control (SCADE V6)



Other Examples

- the cruise control
- the heater
- the (Milner) coffee machine
- approximation methods (Euler, Runge-Kutta)

Laboratory language

Collaboration with the SCADE team since 1999

- the ReLuC compiler of SCADE is based (and improves) techniques introduced in Lucid Synchrone
- typing, clock calculus
- some constructions (e.g., `merge`)
- static analysis (initialization)
- design/semantics of SCADE V6

Collaboration with Athys (Dassault-Systèmes) for the integration of a programming environment into the Catia suite for industrial systems (LCM)

- automatic type synthesis (with polymorphism)
- other type-based analysis

Conclusion and Future Works

Compilation, semantics

- other extensions, program analysis, etc.
- certified compilation (for software and hardware), proof assistant tools

Relaxed Synchrony for Video Systems

- relax (a little) the clock calculus in order to compose non strictly synchronous systems but which can be synchronized through the insertion of buffers
- model of N-Synchronous Kahn Networks [Emsoft'05, POPL'06]
- with the Alchémy project (INRIA) and Philips NatLabs

Take Physical Resources into Account

- how to model real (physical) time, resources?
- how to compile synchrony on a pipelined machine (or a compiled parallel machine)?