Soutien GLA 2020

Preparation au 2ieme Session de l’Examen.

B. Wolff, 7.6.2020

Enoncé

- On reprend l'examen du décembre 2019.
- Link sur le site GLA:

  https://www.lri.fr/~wolff/teach-material/2019-20/L3-GLA/enonce.pdf

Exercice I

Exercise I

“Function qui calcule la multiplication de deux nombres entiers a et b avec b positif ou nul.”

```
mult(int a, int b) {
int res = 0;
while(b > 0) {
    if(b mod 2 != 0) {
        res = res + a;
        b = b - 1;
    }
    a = 2 * a;
    b = b/2;
}
return res;
```

Exercise I 1

“Specification ?”

Pre = b >= 0

Post = result = res ∧ result = a * b

Exercise I 2

“Trois tests fonctionnels juste comme ca.”

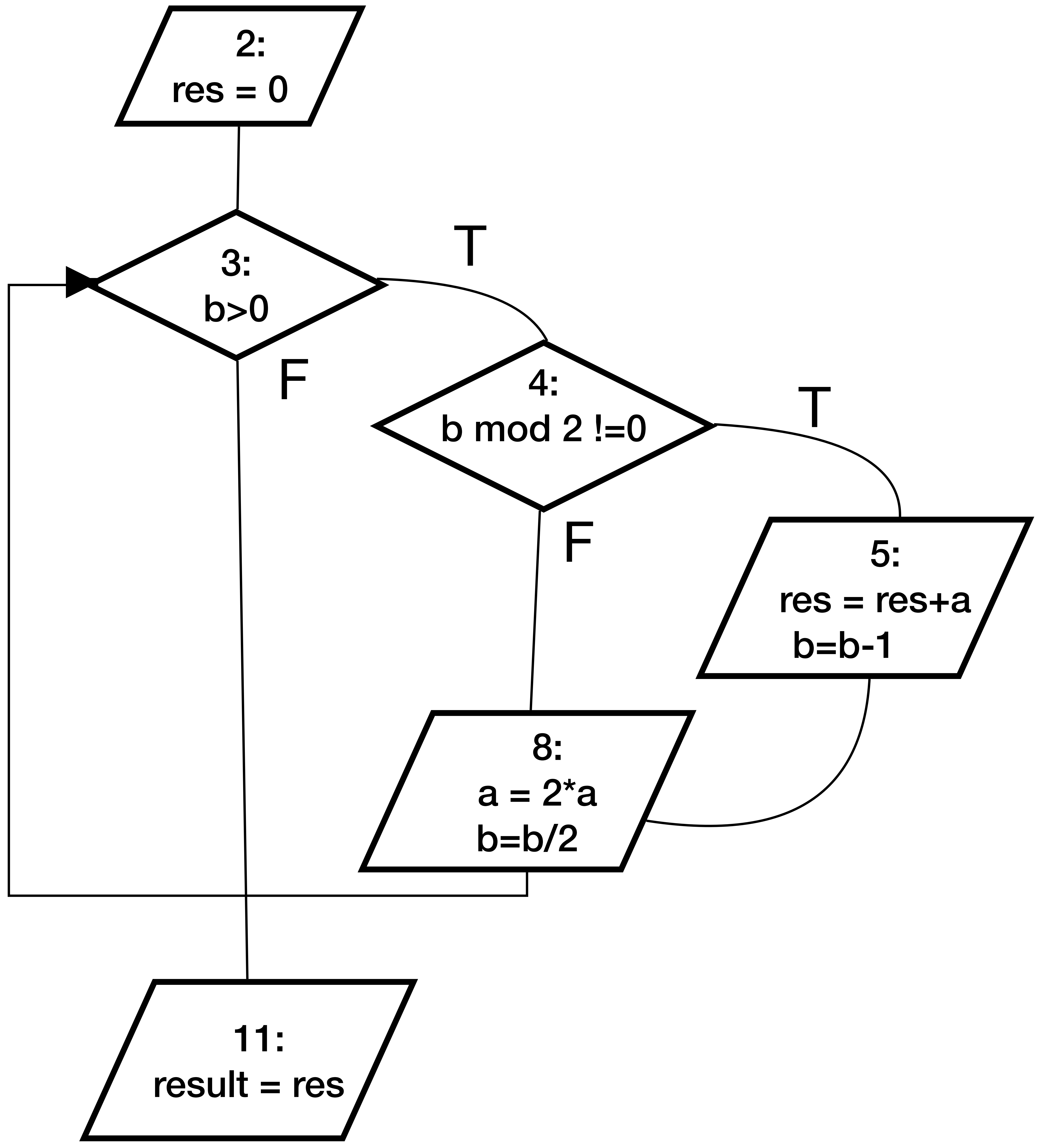
	a	b	result
mult 0	0	0	0
negatif	-2	2	-8
bordercase	1	1	2

Exercise I 3

“CFG ?”

Exercise I 3

“CFG ?”



Exercise I 4

R =   2 (3 4 [5] 8)* 11

“RegExp of CFG ?”

Exercise I 5

TI = {ch1} = {[2,3,4,5,8,11]}

“Toutes instructions”

Exercise I 6

“Exec Symbolique ch1”

	2	3	4	5	8	11
phi	b0>=0	b0>0	b0>0 /\ b0 mod 2 <> 0	“	“	“
a	a0	“	“	a0	2*a0	“
b	b0	“	“	b0 - 1	(b0-1) / 2	“
res	0	“	“	a0	“	“
result	result0	“	“	“	“	a0

Exercise I 6

"Exec Symbolique ch1"

Condition de Chemin ch1 :

$b0>0 \wedge b0 \bmod 2 <> 0$

Exercise I 7

“Faisabilité ch1”

Faisable : OUI

La condition de chemin de ch1 est parfaitement faisable.

Exercise I 8

“Allpath3 (CFG)”

Exercise I 9

“Allpath $_3$ (CFG)”

Allpath$_3$ = {[2,3,11],

[2,3,4,8,11],
[2,3,4,5,8,11],

[2,3,4,8,4,8,11],
[2,3,4,8,4,5,8,11],
[2,3,4,5,8,4,8,11],
[2,3,4,5,8,4,5,8,11],

[2,3,4,8,4,8,4,8,11],
[2,3,4,8,4,5,8,4,8,11],
[2,3,4,5,8,4,8,4,8,11],
[2,3,4,5,8,4,5,8,4,8,11],
[2,3,4,8,4,8,4,5,8,11],
[2,3,4,8,4,5,8,4,5,8,11],
[2,3,4,5,8,4,8,4,5,8,11],

[2,3,4,5,8,4,5,8,4,5,8,11]}

Exercise I9

"Allpath 3 (CFG)"

$f(k) = |\text{Allpath } k| =$

$= \ 1 + 2 + 4 + \ldots$

$= \ 2^0 + 2^1 + 2^2 + \ldots + 2^k$

$= \ 2^{k+1} - 1$

Exercise I 10

“Symbex ch2”

	2	**3**	**11**		
phi	b0>=0	b0>=0 /\ b0<=0	b0 = 0		
res	0	“	“		
a	a0	“	“		
b	b0	“	“		
result	result0	“	0		

Condition de chemin: b0 = 0 faisable !!!

Exercise I 10

“Symbex ch3”

	2	3	4	8	11	
phi	b0>=0	b0 > 0	b0 > 0 ⋀ b0 mod 2 = 0	“	“	
res	1	“	“	“	“	
a	a0	“	“	2*a0	“	
b	a0	“	“	b0 div 2	“	
result	result0	“	“	“	1	

Condition de chemin: $b0 > 0 \land b0 \bmod 2 = 0$ faisable !!!

Exercise I 11

Test-Execution ?

Verdict ?

	a	b	result	verdict res = a0 * b0 ?
ch1	5	1	5	pas d’erreur
ch2	3	0	0	pas d’erreur
ch3	3	4	1	erreur

Exercise II 1

Preuves

(a) $\vdash \{x^3 < x\}$ `x:==x*x` $\{0 < x\}$

(b) $\vdash \{x > 1\}$ `IF x < 8 THEN x :== x*x ELSE x :== 33` $\{x < 53\}$

(c) $\vdash \{X = i! \wedge i > 0\}$ `i :== i+1; X :== X*i` $\{X = i! \wedge i > 0\}$

(d) $\vdash \{x \leq 0\}$ WHILE `x` $\leq$ `0` DO `x :== x+3` $\{1 \leq x \wedge x \leq 4\}$

(e) $\vdash \{1 \leq x\}$ WHILE `x` $<$ `1` DO `x :== x+1` $\{x = 1\}$

(f) $\vdash \{a \geq 0 \wedge a < b \wedge b^2 < c \wedge c \leq a\}$ `c :== b; c :== b*c` $\{c > 12\}$

Exercise II 1a

Preuves

** $x^3 < x \longrightarrow 0<x\ [x \mapsto x^2]$

$\equiv x^3 < x \longrightarrow 0<x^2$

cas 1: $x > 0$

$x^2 < 1 \longrightarrow 0<x^2$

$\equiv \text{False} \longrightarrow 0<x^2 \equiv \text{True}$

cas 2: $x < 0$

$x^2 > 1 \longrightarrow 0<x^2 \equiv \text{True}$

cas 3 : $x = 0$

$x^3 < x \longrightarrow 0<x^2$

$\equiv 0 < 0 \longrightarrow 0<x^2$

$\equiv \text{False} \longrightarrow 0<x^2 \equiv \text{True}$

$$\dfrac{{}^{**}\ x^3 < x \longrightarrow 0<x\ [x \mapsto x^2] \qquad \dfrac{}{\vdash \{0 < x[x \mapsto x*x]\}\ x := x * x\ \{0 < x\}}\ \text{aff} \qquad 0 < x \longrightarrow 0 < x}{\vdash \{x^3 < x\}\ x := x * x\ \{0 < x\}}\ \text{conseq}$$

Exercise II b

Preuves

$$\dfrac{\dfrac{*}{\vdash \{x>1 \land x<8\}\ x := x*x\ \ \{x < 53\}} \qquad \dfrac{**}{\vdash \{x>1 \land x>=8\}\ x := 33\ \{x < 53\}}}{\vdash \{x>1\}\ \text{IF}\ x < 8\ \text{THEN}\ x := x*x\ \text{ELSE}\ x := 33\ \{x < 53\}}\ \text{if}$$

Exercise II b*

Preuves

$$x>1 \land x<8 \longrightarrow (x<53\ [x \mapsto x^2])$$
$$\equiv x>1 \land x<8 \longrightarrow x*x<53$$
$$\equiv \text{True}$$

$$\dfrac{x>1 \land x<8 \longrightarrow x<53\ [x \mapsto x^2] \qquad \dfrac{}{\vdash \{x<53[x \mapsto x^2]\}x := x*x\ \{x < 53\}}\ \text{aff} \qquad x < 53 \longrightarrow x < 53}{\vdash \{x>1 \land x<8\}\ x := x*x\ \{x < 53\}}\ \text{conseq}$$

Exercise II b **

Preuves

$x>1 \land x>=8 \longrightarrow (x<53\ [x \mapsto 33])$

$\equiv\ x>1 \land x>=8 \longrightarrow 33<53$

$\equiv$ True

$$\dfrac{x>1 \land x>=8 \longrightarrow x<53\ [x \mapsto 33] \quad \dfrac{}{\vdash \{x<53[x \mapsto 33]\}\ x := 33\ \{x < 53\}}\text{aff} \quad x < 53 \longrightarrow x < 53}{\vdash \{x>1 \land x>=8\}\ x := 33\ \{x < 53\}}\text{conseq}$$

$$A \equiv X = i! \wedge i > 0[X \mapsto X*i]$$
$$\equiv X*i = i! \wedge i > 0$$

Exercise II 1c

Preuves

** $X=i! \wedge i> 0 \longrightarrow A[i \mapsto i+1]$

$\equiv X=i! \wedge i> 0 \longrightarrow X*i = i! \wedge i > 0[i \mapsto i+1]$

$\equiv X=i! \wedge i> 0 \longrightarrow X * (i+1) = (i+1)! \wedge ii+1 > 0$

$\equiv X=i! \longrightarrow X * (i+1) = (i+1)!$

$\equiv X=i! \longrightarrow i! * (i+1) = (i+1)!$

$\equiv$ True

$$\dfrac{\dfrac{**}{X=i! \wedge i> 0 \longrightarrow A[i \mapsto i+1]} \quad \dfrac{}{\vdash \{A[i \mapsto i+1]\}\ i:=i+1 \{A\}}\text{affect} \quad A \longrightarrow A}{\vdash \{X=i! \wedge i> 0\}\ i:=i+1\ \{X = i! \wedge i > 0[X \mapsto X*i]\}}\text{cons}$$

$$\dfrac{}{\vdash \{X = i! \wedge i > 0[X \mapsto X*i]\}\ X := X*i\ \{X = i! \wedge i > 0\}}\text{affect}$$

$$\dfrac{\vdash \{X=i! \wedge i> 0\}\ i:=i+1\ \{X = i! \wedge i > 0[X \mapsto X*i]\} \quad \vdash \{X = i! \wedge i > 0[X \mapsto X*i]\}\ X := X*i\ \{X = i! \wedge i > 0\}}{\vdash \{X=i! \wedge i> 0\}\ i:=i+1;\ X := X*i\ \{X = i! \wedge i > 0\}}\text{seque}$$

Exercise II 1d

Preuves

Trouve I de sorte que:

a) $x \leq 0 \longrightarrow I$

b) $I \wedge x \leq 0 \longrightarrow I\,[x \mapsto x+3]$

c) $I \wedge x>0 \longrightarrow 1 \leq x \wedge x \leq 4$

est le preuve Hoare est complet.

Solution pour I:

$I \equiv x \leq 4$

et on verifier a) b) c).

$$\dfrac{x \leq 0 \longrightarrow I \qquad \dfrac{\dfrac{I \wedge x \leq 0 \longrightarrow I\,[x \mapsto x+3] \qquad \dfrac{}{\vdash \{I\,[x \mapsto x+3]\}\ x:= x+3\ \{I\}} \qquad I \longrightarrow I}{\vdash \{I \wedge x \leq 0\}\ x:= x+3\ \{I\}}\ \text{conseq}}{\vdash \{I\}\ \text{WHILE}\ x \leq 0\ \text{DO}\ x:= x+3\ \{I \wedge x>0\}}\ \text{while} \qquad I \wedge x>0 \longrightarrow 1 \leq x \wedge x \leq 4}{\vdash \{x \leq 0\}\ \text{WHILE}\ x \leq 0\ \text{DO}\ x:= x+3\ \{1 \leq x \wedge x \leq 4\}}\ \text{conseq}$$

Exercise II e

(a) $\vdash \{x^3 < x\}$ `x:==x*x` $\{0 < x\}$

(b) $\vdash \{x > 1\}$ `IF x < 8 THEN x :== x*x ELSE x :== 33` $\{x < 53\}$

(c) $\vdash \{X = i! \wedge i > 0\}$ `i :== i+1; X :== X*i` $\{X = i! \wedge i > 0\}$

(d) $\vdash \{x \leq 0\}$ WHILE `x` $\leq$ `0` DO `x :== x+3` $\{1 \leq x \wedge x \leq 4\}$

(e) $\vdash \{1 \leq x\}$ WHILE `x` $<$ `1` DO `x :== x+1` $\{x = 1\}$

(f) $\vdash \{a \geq 0 \wedge a < b \wedge b^2 < c \wedge c \leq a\}$ `c :== b; c :== b*c` $\{c > 12\}$

Preuves

C’est faux : Contre-exemple : x = 5.

Exercise II 1f

[redacted]

(f) $\vdash \{a \geq 0 \wedge a < b \wedge b^2 < c \wedge c \leq a\}$ `c :== b; c :== b*c` $\{c > 12\}$

Preuves

Preuve trivial (falseE) parce que :

a >= 0 /\ a < b /\ b < c —> c > a,

donc precondition False.