FIIL PIA Project : Equivalence between Regular Expressions and Nondeterministic Automata with ϵ-transitions for RegExps with Interleave

October 23, 2018

Burkhart Wolff, Master FIIL Université Paris Saclay

Abstract

This project aims at proving completeness and correctness of a compiler converting regular expressions into nondeterministic automata with ϵ-transitions. The objective is to acquire basic knowledge in interactive theopry development in isabelle/HOL; the goal is to complete a skeleton of theories to this end. The work is based on an existing theory — the challenge is to extend it by extendind the regular expression language by the interleave-operator and, via the definition of an adequate representation in terms of automata operations, proving proof that the interleave operation maintains regularity.

Modelization

The theory skeleton consists of the following components:

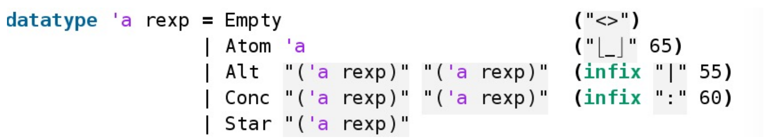
1. `RegExp.thy` contains a the abstract syntax of regular expression together with a *denotational semantics*, a function L that assigns to each regular expression r the "language" it denotes: $L(r)$.

2. `Automata.thy` defines non-deterministic and deterministic automata, the generalized transition functions as well as the corresponding acceptance conditions for a word in the automata.

3. `RegExp2NAe.thy` contains the compiler and its corresponding correctness nad completeness conditions.

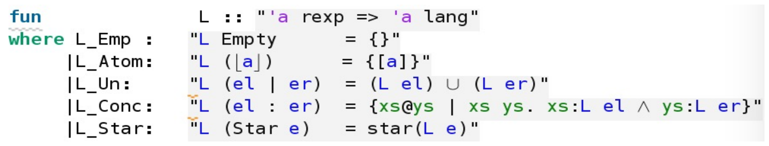
Regular Expression

There is a **abstract syntax of regular expression** and a function L give the language of this regular expression.



```
datatype 'a rexp = Empty                              ("<>")
                 | Atom 'a                            ("⌊_⌋" 65)
                 | Alt  "('a rexp)" "('a rexp)"  (infix "|" 55)
                 | Conc "('a rexp)" "('a rexp)"  (infix ":" 60)
                 | Star "('a rexp)"
```

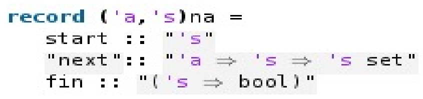
A function L give the language of this regular expression.



```
fun              L :: "'a rexp => 'a lang"
where L_Emp :   "L Empty      = {}"
     |L_Atom:   "L (⌊a⌋)      = {[a]}"
     |L_Un:     "L (el | er)  = (L el) ∪ (L er)"
     |L_Conc:   "L (el : er)  = {xs@ys | xs ys. xs:L el ∧ ys:L er}"
     |L_Star:   "L (Star e)   = star(L e)"
```

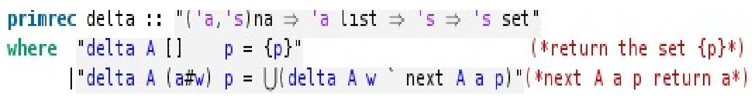
Nondeterministic automata

na represent nondeterministic automata. It is defined In the theory Automata using record.



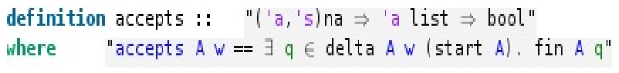
```
record ('a,'s)na =
   start :: "'s"
   "next":: "'a ⇒ 's ⇒ 's set"
   fin :: "('s ⇒ bool)"
```

Delta to calcul a set of states reachable after a word in term of w from initial states.



```
primrec delta :: "('a,'s)na ⇒ 'a list ⇒ 's ⇒ 's set"
where  "delta A []    p = {p}"                    (*return the set {p}*)
      |"delta A (a#w) p = ⋃(delta A w ` next A a p)"(*next A a p return a*)
```

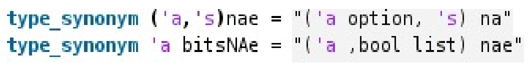
Accepts A Definition decide if a word ca be accepted by nondeterministic automata na which is defined here.



```
definition accepts ::   "('a,'s)na ⇒ 'a list ⇒ bool"
where      "accepts A w == ∃ q ∈ delta A w (start A). fin A q"
```

Nondeterministic automata with epsilon transitions

nae is defined for nondeterministic automata with epsilon transitions, which is defined in RegExp2DAe.thy. nae is an instance of the nondeterministic (in)finite automaton(na), which allows a transformation to a new state without consuming any input symbols.



```
type_synonym ('a,'s)nae = "('a option, 's) na"
type_synonym 'a bitsNAe = "('a ,bool list) nae"
```

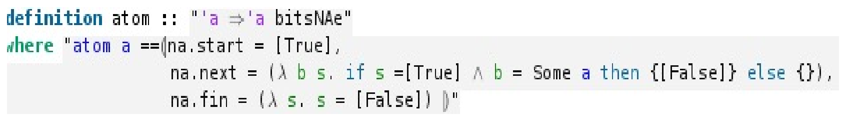
From regular expressions directly to nondeterministic automata with epsilon

The transition function rexp2nae converts regular expressions into such automata and is defined in file `RegExp2DAe.thy`. It use 5 definitions for the **Atom Conc Union Star** case which are defined in the same file. The goal of this project is showing the correctness of this translation.

A key-problem of the construction is that the nodes from which automatas were constructed must be disjoint when "gluing" several automatas together; the present modelization realises this by a re-labelling discipline on node-names which were represented as bitstrings.

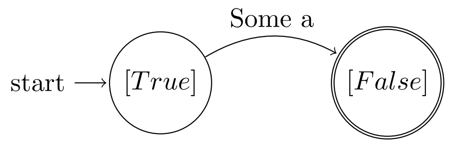
We present some diagrams to illustrate transition function of these automatas.

Atom: The Atom case. In its initial state [True] , it may consume Some a (of any type), and go over to accepting state [False] . Here is the formal definition:

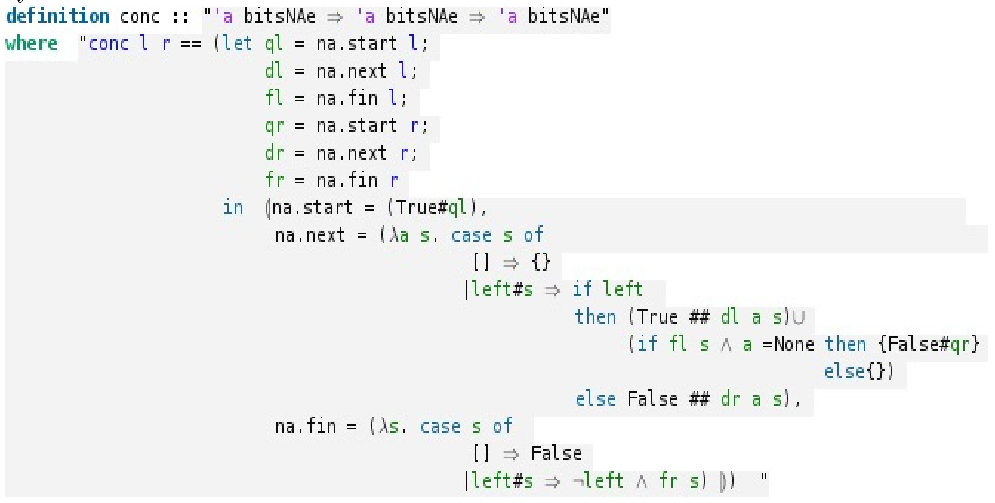


```
definition atom :: "'a ⇒'a bitsNAe"
where "atom a ==⦇na.start = [True],
                na.next = (λ b s. if s =[True] ∧ b = Some a then {[False]} else {}),
                na.fin = (λ s. s = [False]) ⦈"
```

... reflecting the automaton:

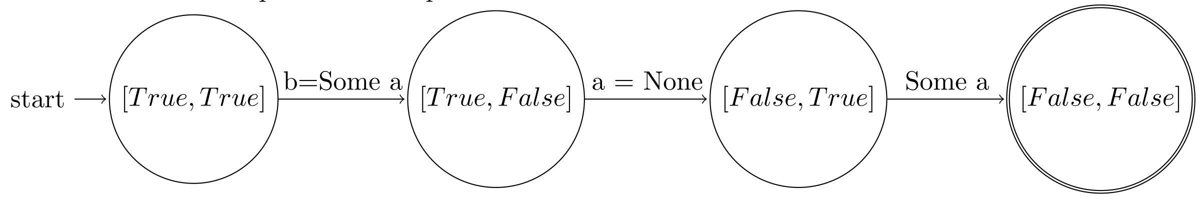


Conc: The conc case. It takes two NAe l and r and returns a new NAe. What is interesting here is the injective re-labelling of the nodes; any node label in the left automaton is prefixed by `True`, any node label in the right automaton is prefixed by `False`. Since the translation of regular expressions will be done recursively over the structure of regular expressions, the construction assures that the node sets are always distinct.



```
definition conc :: "'a bitsNAe ⇒ 'a bitsNAe ⇒ 'a bitsNAe"
where  "conc l r == (let ql = na.start l;
                         dl = na.next l;
                         fl = na.fin l;
                         qr = na.start r;
                         dr = na.next r;
                         fr = na.fin r
                     in  ⦇na.start = (True#ql),
                          na.next = (λa s. case s of
                                           [] ⇒ {}
                                          |left#s ⇒ if left
                                                    then (True ## dl a s)∪
                                                         (if fl s ∧ a =None then {False#qr}
                                                                            else{})
                                                    else False ## dr a s),
                          na.fin = (λs. case s of
                                           [] ⇒ False
                                          |left#s ⇒ ¬left ∧ fr s) ⦈)  "
```

Here is the example of the concatenation of two automata l and automata r which happen to be the atomic automaton of the previous example.



Tasks:

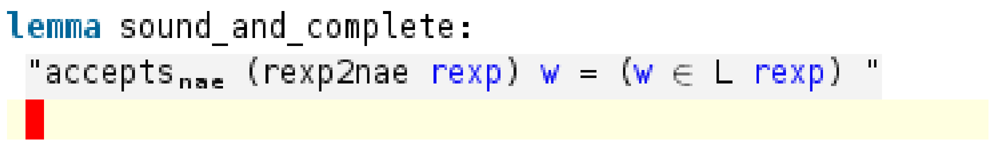
1. Study the theory in Isabelle.

2. Fill in the holes (sorry's) in the various levels of proofs, in particular soundness and completeness of the translation, reflected by the theorem, that all accepted words of the translation were contained in the language of the original regular expression.

3. add the interleave operation Interleave `||` in the `regexp`-syntax. (If you need to simplify: You may assume that `||` occurs only in outermost positions in a `regexp`.)

4. The semantics of this operation is already denotationally defined.

5. define the translation ans prove correctness and soundness.



```
lemma sound_and_complete:
 "accepts_nae (rexp2nae rexp) w = (w ∈ L rexp) "
```

Submission

The final version of the files together with a 3-6 pages report of the theory development are due **monday, 21.1.2019** per mail at `wolff@lri.fr`.