Master FIIL : Preuve interactives et Applications

Année 2018-2019

Prof. Burkhart Wolff
wolff@lri.fr

TP 1 - Introduction to Isabelle/HOL

Semaine du 17 septembre 2018

Exercice 1 (Installation)
Install Isabelle(version 2018) from the Isabelle website `https://isabelle.in.tum.de`. Start Isabelle and make yourself familiar with the documentation, in particular the overview "What's in Main" where "Main" is the standard HOL library we base our exercises on.

Exercice 2 (Editing, Type-Checking, Searching)
Use the Isabelle commands `prop`, typ+ and term+ to enter types and terms into the system, thus using parser and type-checker of Isabelle.

Questions

1. Enter, parse and type-check (if possible) the term

   $$(\lambda x.\lambda y.(\lambda z.(\lambda x.zx)(\lambda y.zy))(xy))$$

   (It might be helpful to add spaces ...) Note how the system represents bound and free variables.
2. Define via a number of `definition`s the Church Numerals of the slides of class 1, page 16 ff.

   Which type do Church-Numerals have in the typed λ-calculus ?
3. axiomatize the Y-combinator. The syntax is :

   $$\text{axiomatization} \quad const\_name \quad :: \quad typ \quad \text{where} \quad ax\_name : \quad ''eqn''$$

   Which type has to be given to the Y-combinator for this axiomatization ?
4. use `find_theorems` to browse your theory so far !
5. Prove that, according to your definitions, $2 + 3$ is indeed 5.
   Hint : state a `lemma` for this equation, unfold the definitions, and apply either reflexivity or the simplification method `simp`

Exercice 3 (Simple Induction Proofs)
Prove

$$2 * (\Sigma i = 0..(n :: nat).i) = n * (n + 1)$$

in HOL.

Hints :

1. search for an appropriate Induction scheme in the theory `Nat`
2. apply it as suitably instantiated (substitution !) rule via the variant `rule_tac`
3. apply simplifications.

Why is the `::nat` necessary ? Why is the rule instatiation necessary ?